Endowment Structure, Industrial Dynamics, and Economic Growth

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Abstract

This paper develops a dynamic general equilibrium model to explore industrial evolution and economic growth in a closed developing economy. We show that industries will endogenously upgrade toward the more capital-intensive ones as the capital endowment becomes more abundant. The model features a continuous inverse-V-shaped pattern of industrial evolution driven by capital accumulation: As the capital endowment reaches a certain threshold, a new industry appears, prospers, then declines and finally disappears. While the industry is declining, a more capital-intensive industry appears and booms, ad infinitum. Explicit solutions are obtained to fully characterize the whole dynamics of perpetual structural change and economic growth. Implications for industrial policies are discussed.

Key Words: Endowment, Industrial Dynamics, Economic Growth, Structural Change

JEL Codes: L50, O14, O40

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"The growth of GDP may be measured up in the macroeconomic treetops, but all the action is in the microeconomic undergrowth, where new limbs sprout, and dead wood is cleared away."

–*World Bank Commission on Growth and Development* (2008, pp.2-3)

1 Introduction

The main goal of this paper is to develop a formal model to explain the industrial dynamics along the path of economic growth in developing countries. We show how the optimal leading industries are structurally different at different development stages, depending mainly on the economy's endowment structure and its evolution. Sustained economic development from a low-income status to high-income status in countries in modern times is characterized by continuous technological innovation and industrial upgrading (see Chenery (1960), Kuznets (1966), Maddison (1980), Chenery, Robinson, and Syrquin (1986), Hayami and Godo (2005)). Beneath the GDP growth, the products or major industries in the manufacturing sector of these economies are continuously changing over time. First labor-intensive goods such as textiles and shoes are produced, then those industries decline and are gradually replaced by the more capital-intensive industries such as machinery and electronics, which also decline later while even more capital-intensive industries arise such as cars and aircraft, and so forth. Such a pattern, as shown in Figure 1, was referred as the flying geese pattern of economic development by Akamatsu (1962) in the 1930s and further developed by Kojima (2000).

[Insert Figure 1 Here]

Surprisingly, however, this continuous waxing and waning pattern of industrial development has rarely been formalized in the growth and development literature, although it has been well documented for a long time.[1] Recall that most of the

[1] There are some exceptions: Vernon (1965) develops a product-cycle argument to explain how the location of production for a commodity might shift across countries over time. Schumpeter

earlier growth models aim to match the Kaldor facts and typically assume the same aggregate production function for countries at different development stages, which naturally leads economists to focus on the cross-country differences in productivity or human capital while ignoring the structural differences in the industries for countries at different development stages (see Kaldor (1961), Solow (1965), Barro and Sala-i-Martin (2004)).[2] Recent growth models do start to address various types of structural change. Most of them mainly explore the long-run trend shift in the compositions of aggregate agriculture, industry, and service sectors without exploring the dynamics within the aggregate sectors, such as the continuous upgrading of manufacturing industries. Consequently, they do not characterize the inverse-V-shaped industrial dynamics described above. For example, Lucas (2004) studies the rural-urban transformation driven by the externality of human capital. Buera and Kaboski (2009) focus on the expansion of the service sector. Some other works strive to match Kuznets facts, which state that development is typically a process of a decline in agriculture, a rise in services, and a hump-shaped change in industry. In addition, that literature mainly focuses on the long-run (balanced or asymptotic) growth rate (or steady state) without explicitly and completely characterizing the whole dynamics for the structural change *per se*. Moreover, in those models the structural changes are driven either by the demand shift in the consumption goods as people get richer (see Laitner (2000), Caselli and Coleman (2001), Kongsamut, Rebelo, and Xie (2001), Gollin, Parente, and Rogerson (2002)), or by the different productivity growth across different sectors as first suggested by Baumol (1967), and further developed by Hansen and Prescott (2002) and Ngai and Pissarides (2007). In this paper, we argue that the inverse-V-shaped industrial dynamics in a developing country is driven mainly by the change in its endowment structure.

(1942) expatiates the idea of creative destruction based on technology advancement instead of capital endowment improvement, which is further developed by Aghion and Howitt (1992). Those studies focused mainly on the R&D-driven mechanism of industrial evolution in developed countries instead of developing countries.

[2] Kaldor facts refer to the relative constancy of the growth rate of total output, the capital-output ratio, the real interest rate, and the share of labor income in GDP.

The endowments are given at any given time and changeable over time. One of the key differences between a developed and developing country is the difference in the relative abundance of capital in their endowment structures. The economic development process in a developing country is characterized by the continuous upgrading of its endowment structure from relatively scarce in capital and relatively abundant in labor or natural resources to relatively abundant in capital and relative scarce in labor/natural resources. Lin (2003, 2009) argues that the optimal industrial structure in an economy at a given time should be consistent with the given endowment structure at that time: as capital accumulates and becomes relatively cheaper, the industries should optimally upgrade toward more capital-intensive ones accordingly. Motivated by Lin's argument, our model will show that the driving force for the flying-geese pattern of industrial upgrading in the development process of a developing country is the continuous capital deepening in the endowment structure.

Acemoglu and Guerrieri (2008) have examined how the capital deepening has an asymmetric impact on the sectors with different capital intensities, but their main objective is to study how the elasticity of substitution between two sectors with different capital shares affects the long-run asymptotic aggregate growth rate. Moreover, their model has two sectors and is thus unable to explain the flying-geese pattern of repetitive inverse-V-shaped industrial upgrading dynamics.

It is very important to understand the widely observed inverse-V-shaped industrial dynamics in the process of economic development. What type of industry should we expect to dominate at a certain stage of development? Would it be optimal for the government in a low-income country to support the development of certain industries that prevail in high-income countries? To answer these questions, it is not enough to merely recognize the long-run structural change among the primary, secondary, and tertiary sectors or in the rural-urban transformation.

In what follows, we develop a growth model featuring this inverse-V-shaped industrial dynamics along the development path. The major force driving the structural change is the increase in the capital-labor ratio (or alternatively, endowment structure). As capital becomes more abundant and relatively cheaper, the more capital-intensive industrial goods are produced, because the more capital-intensive industry products are not only more affordable but also more desirable for the consumers. At the same time, the more labor-intensive goods are gradually displaced. As capital becomes even more abundant, goods with an even higher capital intensity become more desirable to produce and consume. This generates the endless inverse-V-shaped industrial dynamics. Our model underscores the key role played by the changes in endowment structure rather than productivity increase. This might be reasonable because our model is mainly geared toward developing economies where industrial upgrading relies mainly on borrowing existing technologies from developed countries (Hayami and Goto (2005)), in contrast to the developed economies where huge research and development expenditure is required for technology innovation and industrial upgrading.

To highlight the industrial dynamics, we deviate from the standard practice in the growth literature, which typically only focuses on the steady state or the long-run (balanced or asymptotic) growth rate. Instead, we obtain the explicit solution for the whole dynamics in the structural changes, even though we are considering a dynamic economy with an infinitely dimensional commodity space and an infinite time horizon.[3] We decompose this seemingly complicated structural analysis into two steps. First, along the time dimension, the representative household simply decides the allocation of capital for producing consumption goods and savings. The household's capital allocation determines the evolution of capital endowment and the intertemporal use of capital in production. Then, along the cross-section dimension, the capital allocated for production determines the production structures (industrial

[3] There may be a technical reason why the well-recognized fact of industrial dynamics has rarely been formalized in growth models. Closed-form solutions for transitional dynamics are typically very hard to obtain even in most of the two-sector growth models, but the aforementioned inverse-V-shaped industrial dynamics is by nature the transitional dynamics per se, so the industrial upgrading with an infinite-dimensional commodity space appears even more unwieldy.

choices) as if it were a static model. Ultimately, the mathematical problem is reduced to dynamic optimization with switching state equations. This approach, which we call a dynamic structural analysis, can tremendously simplify the analysis of perpetual industrial upgrading with long-run growth.

Our paper is related to the literature of quality-ladder growth models because in our model different industrial goods are modelled as perfect substitutes and are produced with different technologies. Aghion and Howitt (1992) emphasize that the firm's incentives to earn monopolistic rents justify its endeavor to undertake risky and costly R&D, which ultimately causes the creative destruction (also see Grossman and Helpman, 1991a). Our model is methodologically closest in spirit to Stokey (1988), who characterizes how the learning-by-doing keeps the band of the produced commodities moving toward higher and higher qualities. The main goals of that literature, however, are to generate sustained economic growth at the aggregate level instead of trying to explain the inverse-V-shaped industrial dynamics. More importantly, all these papers try to emphasize the role of technological advancement or knowledge accumulation with stable industries while our paper stresses the role of capital accumulation and industrial upgrading. In addition, this literature, like other growth models, also mainly studies the long-run balanced growth path while leaving the transitional dynamics aside. The industrial climbing result in Stokey (1988), for example, is obtained essentially through comparative statics rather than the full-blown dynamic model. By contrast, we are able to provide the closed-form solutions for all the dynamics.

Simple comparative statics in our static model shows how the change in the endowment structure determines the changes in industrial structures, a result similar to the Heckscher-Ohlin model with multiple diversification cones (see Leamer (1987), Schotter (2003)). However, there is a nontrivial difference. These multiple diversification cone models mainly consider open economies where the production structure of a country is determined by international specialization. By contrast, in our closed economy model, the households and firms select endogenously which set of products to consume and produce. More importantly, our dynamic model enables us to obtain explicit solutions to characterize the whole inverse-V-shaped dynamics of the infinite industrial upgrading, while H-O models with multiple diversification cones are mostly static. To highlight the direct impact of endowment change on the industrial dynamics, we purposefully ignore the effect of international specialization according to comparative advantage and only consider a closed economy in this paper.[4] We suspect that our main proposition, namely, the change in endowment structures drives the change in industrial structures, will be only strengthened in an open economy, as predicted by standard H-O trade models.[5] Our model characterizes the first-best scenario in which the industrial structures evolve optimally in a perfectly competitive and frictionless economy. In such an ideal world, no government intervenes and the market itself can identify and support the right industries at each development stage. But what if the government pursues a wrong development strategy and pushes the economy to develop some inappropriate industries? We show in this paper that such policy mistakes may sometimes cause the economy to fall into a poverty trap such that long-run growth becomes impossible without foreign help. The markets are far from perfect in the real world, so it may be desirable for the government to have an industrial policy which will help to provide firms with information, coordinate firms' investments and compensate for externalities produced by pioneer firms (Murphy, Shleifer, and Vishny (1989), Lin (2009)). However,

[4] Notice that in our paper a developing country is implicitly assumed to be able to freely borrow the technology knowhow of more capital intensive industries from the developed countries. The main conclusions of the model are expected to hold in an open economy model. Moreover, except in an extremely small economy, the domestic market plays a major role in economic development. For example, Chenery, Robinson, and Syrquin (1986) find that expansion in the domestic demand accounted for a 72%-74% increase in domestic industrial output in those countries with population larger than 20 million, and that even for small and manufacturing-oriented countries with population less than 20 million, domestic demand expansion accounts for a 50%-60% increase in the total industrial output. See Murphy, Shleifer and Vishy (1989) for more argument.

[5] Ventura (1997) shows theoretically how factor-price-equalization driven by international trade can explain the rapid catching-up growths of several export-oriented East Asian economies. Krugman (1979) constructs a North-South trade model to explore how the catching up process depends on whether the rich country's innovation speed exceeds the poor country's imitation speed.

the prerequisite for a successful industrial policy is to identify what type of industries should be supported at each different development stage. Our first-best characterization sets a theoretical benchmark that may potentially help us think further about these issues.

The paper is organized as follows: Section 2 presents the static model. The dynamic model is analyzed in Sections 3 and 4. Welfare consequences of the mistakes in industrial choices are discussed in Section 5. Section 6 concludes. Technical proofs and derivations are put in the Appendix.

2 Static Model

2.1 Setup

Consider a closed developing economy with a unit mass of identical households. Each household is endowed with L units of labor and E units of capital, which conceptually consists of both tangible physical capital and intangible capital. For parsimony purpose, we will narrowly interpret this "composite capital" as physical capital from now on in this qualitative investigation. The given commodity space has infinite dimensions. Let c_n denote the consumption of good $n = 0, 1, 2, ...$ In particular, good 0 may be interpreted as a household product, and good n for $n \geq 1$ may be interpreted as “industrial” product. Define the aggregate good as

$$C = \sum_{n=0}^{\infty} \lambda_n c_n,$$

where the quality coefficient for good i is λ_n.[6] We require $c_n \geq 0$ for any n. The representative household's utility function is $CRRA$:

$$U = \frac{C^{1-\sigma} - 1}{1-\sigma}, \text{ where } \sigma \in (0, 1]. \tag{1}$$

All the production technologies exhibit constant returns to scale. In particular, good 0 is produced with labor only. One unit of labor produces one unit of good 0. For any industrial good $n = 1, 2, 3, ...$, both labor and capital are required and the production functions are Leontief: [7]

$$F_n(k, l) = \min\{\frac{k}{a_n}, l\}, \tag{2}$$

where a_n is the capital intensity of producing one unit of good n. All the markets are perfectly competitive. Let p_n denote the price of good n. Let r denote the rental price of capital and w denote the wage rate. Thus, firm's zero profit function implies that $p_0 = w$ and $p_n = w + a_n r$ for $n = 1, 2, 3, ...$

We assume

$$\lambda_n = \lambda^n, \; a_n = a^n \tag{3}$$

$$\lambda > 1, \; a > 1, \text{ and } a - 1 > \lambda. \tag{4}$$

[6] This assumption is quite standard in the vertical innovation growth literature. The specification is also mathematically isomorphic to the following alternative economic interpretation: C is the final good while all the $c_n, n = 0, 1, 2, ...$ are intermediate goods, so λ_n should be interpreted as the "productivity" for good n. It is not unusual in growth literature to assume perfect substitutability for the output across different production activities. For example, the agricutural Malthus production and the modern Solow production are two linearly additive components for the total output in Hansen and Prescott (2002) and Lucas (2009). A further discussion will be devoted to this "perfect substitutability" assumption later.

[7] This assumption drastically simplifies the dynamic strucutral analysis partly by giving us a lot of linearities. In the Appendix, we show that the main results remain valid with Cobb-Douglass function, but the dynamic analysis will be much more complex. Houthakker (1956) shows that Leontief production functions with Pareto-distribution heterogenous paratermers can aggregate into Cobb-Douglass production functions. Lagos (2006) constructs another distribution that can aggregate heterogenous Leontief functions into CES production functions. These may be helpful in understanding how firm heterogeneities may affect our results, which is a very interesting research direction.

Therefore, a higher-index good has a higher quality but is more capital intensive. The last inequality in (4) not only rules out the trivial case that only the highest quality good is produced in any equilibrium, but also simplifies our analysis, as will be clear shortly.

The household problem is to maximize (1) subject to the following budget constraint

$$\sum_{n=0}^{\infty} p_n c_n = wL + rE \tag{5}$$

Set up the Lagrangian with the multiplier denoted by μ, and we obtain the following optimality condition for consumptions:

$$\lambda^n \left(\sum_{n=1}^{\infty} \lambda^n c_n + c_0 \right)^{-\sigma} \leq \mu p_n, \text{ for } \forall n \geq 0, \tag{6}$$

$$\text{“} = \text{”} \text{ when } c_n > 0.$$

2.2 Market Equilibrium

The market equilibrium is determined by the endowment structure (capital per capita), $\frac{E}{L}$. In the Appendix, we show that at most two goods are produced simultaneously in the equilibrium and that these two goods have to be adjacent in capital intensities. The intuition is the following: Suppose goods n and $n+1$ are produced for some $n \geq 1$. From consumer's maximization problem, we immediately have

$$MRS_{n+1,n} = \lambda = \frac{p_{n+1}}{p_n} = \frac{w + a_{n+1}r}{w + a_n r},$$

which implies that

$$\frac{r}{w} = \frac{\lambda - 1}{a^n(a - \lambda)}. \tag{7}$$

Obviously, $MRS_{j,j+1} > \frac{p_j}{p_{j+1}}$ whenever $\frac{r}{w} > \frac{\lambda-1}{a^j(a-\lambda)}$, and $MRS_{j,j-1} > \frac{p_j}{p_{j-1}}$ whenever $\frac{r}{w} < \frac{\lambda-1}{a^{j-1}(a-\lambda)}$ for any $j = 1, 2, ...$ Therefore, when $\frac{r}{w} = \frac{\lambda-1}{a^n(a-\lambda)}$, good $n+1$ must be strictly preferred to good $n+2$, because the marginal rate of substitution is larger than their relative price. This means that $c_j = 0$ for all $j \geq n+2$. Using the same logic, we can also verify that $c_j = 0$ for all $1 \leq j \leq n-1$. In addition, condition (4) ensures that good 0 will not be produced, as can be verified by simply comparing the marginal rate of substitution between good n and good 0 and their price ratio. Similarly, when goods 0 and 1 are produced, we can show that good 1 is strictly preferred to any good $n \geq 2$.

The market clearing conditions for labor and capital are

$$c_n + c_{n+1} = L \tag{8}$$

$$c_n a^n + c_{n+1} a^{n+1} = E. \tag{9}$$

[Insert Figure 2 Here]

The market equilibrium can be illustrated graphically in Figure 2, where labor and capital are represented by horizontal and vertical axes, respectively. O represents the origin for the country. Lines $Oa^n = (1, a^n)\, c_n$ and $Oa^{n+1} = \left(1, a^{n+1}\right) c_{n+1}$ are the vectors of factors used in producing c_n and c_{n+1} in the equilibrium. Let point $W = (L, E)$ be the factor endowment of the country. If $W = a^n L$, only good n is produced. Similarly, if $W = a^{n+1}L$, only good $n+1$ is produced. When $a^n L < W < a^{n+1}L$, both goods n and $n+1$ are produced. The factor market clearing conditions, (8) and (9), determine the usages of labor and capital in industries n and $n+1$, which are represented by vector OA and vector OB in the parallelogram $OAWB$, respectively. If the capital endowment increases from W to W', the new equilibrium becomes parallelogram $OA'W'B'$ so that c_n decreases but c_{n+1} increases.

More precisely, the equilibrium output of each commodity c_n, the relative factor prices $\frac{r}{w}$, and the corresponding aggregate output C are summarized in the following table.

Table 1: Static Equilibrium

$0 \leq E \leq aL$	$a^n L \leq E < a^{n+1}L$ for $n \geq 1$
$c_0 = L - \frac{E}{a}$	$c_n = \frac{La^{n+1}-E}{a^{n+1}-a^n}$
$c_1 = \frac{E}{a}$	$c_{n+1} = \frac{E-a^n L}{a^{n+1}-a^n}$
$c_j = 0$ for $\forall j \neq 0, 1$	$c_j = 0$ for $\forall j \neq n, n+1$
$\frac{r}{w} = \frac{\lambda-1}{a}$	$\frac{r}{w} = \frac{\lambda-1}{a^n(a-\lambda)}$
$C = L + (\lambda - 1)\frac{E}{a}$	$C = \frac{\lambda^{n+1}-\lambda^n}{a^{n+1}-a^n}E + \frac{\lambda^n(a-\lambda)}{a-1}L$
$\Leftrightarrow E_{0,1} = \frac{a}{\lambda-1}(C - L)$	$\Leftrightarrow E_{n,n+1} = \left[C - \frac{\lambda^n(a-\lambda)}{a-1}L\right]\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}$

The whole static equilibrium is summarized verbally in the following proposition.

Proposition 1 *In a closed economy, the static market equilibrium is determined by the endowment structure, $\frac{E}{L}$. Generically, only two goods adjacent in capital intensities are produced in equilibrium. As capital per capita increases, every commodity exhibits an inverse-V-shaped life cycle. A good enters the market, prospers (its output increases) and then declines, and finally is fully replaced by another product with a higher capital intensity. The ratio of the interest rate to the wage rate declines as $\frac{E}{L}$ increases.*

Figure 3 shows that the relative factor prices $\frac{r}{w}$, which declines in a stair- shaped fashion as E increases. This discontinuity results from the Leontief production assumption but the flat part is more general: During the structural change, resource reallocation occurs without changes in relative prices.

[Insert Figure 3]

It is straightforward to compute the capital share in the total output, which equals $\frac{\left(\frac{\lambda-1}{a-1}\right)E}{\frac{\lambda-1}{a-1}E+\frac{a^n(a-\lambda)}{a-1}L}$ when $E \in [a^n L, a^{n+1}L]$ for any $n \geq 1$. So the capital share monotonically increases with capital during each diversification cone and then suddenly drops to $\frac{(\lambda-1)}{a-1}$as the economy enters a different diversification cone, but it always falls into the following interval $[\frac{\lambda-1}{a-1}, \frac{(\lambda-1)a}{(a-1)\lambda}]$ for any development stage with $n \geq 1$. This result is consistent with the Kaldor fact that the capital share is fairly stable over time (see Barro and Sala-i-Martin (2004) for more discussions on the robustness of Kaldor facts).

How c_n changes has already been encoded in Figure 2 but its inverse-V-shaped pattern can be more intuitively seen in Figure 4. When capital endowment E reaches threshold value $a^n L$, good $n+1$ enters the market, and its output increases as E increases up to the point $E = a^{n+1}L$ and then it declines. At the point $E = a^{n+2}L$, good $n+1$ exits from the market while a new good with a higher quality $(n+3)$ appears.

[Insert Figure 4]

The above equilibrium in our closed economy model turns out to be very similar to the H-O trade model with multiple diversification cones. However, the main mechanisms are totally different. Leamer (1987) and other papers in this literature mainly consider (small) open economies where the production structure of a country is determined by international specialization and each good has to be consumed. In our closed-economy general equilibrium model, which set of goods should be consumed and produced is an endogenous decision, depending solely on the domestic demand and endogenous relative factor prices, which are mainly dictated by the endowment structure.

3 Dynamic Model

In this section, we will develop a dynamic model to capture the complete industrial dynamics along the path of economic growth. The key idea is to break down the evolution analysis of production structures into two steps. Along the time dimension, the representative consumer decides her intertemporal consumption flows of the aggregate good C and makes the saving and investment decisions. This dynamic decision determines the evolution of endowment structure $\frac{K}{L}$ and the optimal capital expenditure $\frac{E}{L}$ at every time point t. In the cross section dimension, the capital expenditure $\frac{E}{L}$ then determines the production structures, exactly the same as in the static model.

By the second Welfare Theorem, we can characterize the competitive equilibrium by resorting to the following social planner problem:

$$\max_{C(t)} \int_0^\infty \frac{C(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

subject to

$$\dot{K} = \xi K(t) - E(C(t)) \tag{10}$$

$$K_0 \text{ is given.}$$

where ρ is the time discount rate. $K(t)$ is the amount of working capital at t. At each time, the old capital can be transformed into new working capital using the standard AK model technology and ξ is the exogenous technology parameter net of the depreciation rate. All the new working capital can be used to either produce the consumption good or to save/invest. $E(C(t))$ is the total capital used to produce the aggregate consumption $C(t)$. All the consumption goods are non-storable. The total labor endowment L is constant over time.[8] To ensure positive consumption growth, we assume $\xi - \rho > 0$. To exclude the explosive solution, we also assume $\frac{\xi-\rho}{\sigma}(1-\sigma) < \rho$. Putting them together, we assume

$$0 < \xi - \rho < \sigma\xi. \tag{11}$$

From Table 1, we know that $E(C)$ is a strictly increasing, continuous, piece-wise linear function of C. It is not differentiable at $C = \lambda^i L$, for any $i = 0, 1, ...$

[8] It is straightforward to examine how exogenous changes in the "effective labor" (for example, let $L(t) = L_0 e^{\gamma t}$ for some $\gamma > 0$) may affect economic dynamics.

Therefore, the above dynamic problem may involve changes in the functional forms of the state equation: (10) can be explicitly rewritten as

$$\dot{K} = \begin{cases} \xi K, & \text{when} \quad C \leq L \\ \xi K - E_{0,1}(C), & \text{when} \quad L \leq C \leq \lambda L \\ \xi K - E_{n,n+1}(C), & \text{when} \quad \lambda^n L \leq C \leq \lambda^{n+1} L, \text{ for } n \geq 1 \end{cases},$$

where $E_{n,n+1}(C)$ is defined in Table 1 for any $n \geq 0$. We can easily verify that, in this dynamic optimization problem, the objective function is strictly increasing, differentiable and strictly concave while the constraint set forms a continuous convex-valued correspondence, hence the equilibrium must exist and also be unique.

Let t_0 denote the last time point when the aggregate consumption equals L, and t_n denote the first time point when $C = \lambda^n L$ for $n \geq 1$. As can be shown later, the aggregate consumption C is monotonically increasing over time in the equilibrium, hence the problem can be also written as

$$\max_{C(t)} \int_0^{t_0} \frac{C(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt + \sum_{n=0}^{\infty} \int_{t_n}^{t_{n+1}} \frac{C(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

subject to

$$\dot{K} = \begin{cases} \xi K & \text{when} \quad 0 \leq t \leq t_0 \\ \xi K - E_{0,1}(C), & \text{when} \quad t_0 \leq t \leq t_1 \\ \xi K - E_{n,n+1}(C), & \text{when} \quad t_n \leq t \leq t_{n+1}, \text{ for } n \geq 1 \end{cases},$$

K_0 is given.

According to Table 1, in time period $t_0 \leq t \leq t_1$, goods 0 and 1 are produced and $E(C) \equiv E_{0,1}(C) = \frac{a}{\lambda-1}(C - L)$, while in time period $t_n \leq t \leq t_{n+1}$ for $n \geq 1$, goods n and $n+1$ are produced. $E(C) \equiv E_{n,n+1}(C) = \left[C - \frac{\lambda^n(a-\lambda)}{a-1} L\right] \frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}$. If K_0 is smaller than a certain threshold value (to be discussed soon), then there exists a time period $0 \leq t \leq t_0$ in which only good 0 is produced and all the working capital is saved for the future, so that $E = 0$ when $0 \leq t \leq t_0$. If K_0 is large, on the other hand, the economy may start with producing good h and $h+1$ for some $h \geq 1$, so $t_0 = t_1 = \cdots = t_h = 0$ in the equilibrium.

To solve the above dynamic problem, following Kamien and Schwartz (1991), we set the *discounted-value* Hamiltonian in the interval of $t_n \leq t \leq t_{n+1}$, and use subscripts "$n, n+1$" to denote all variables in this interval:

$$\begin{aligned} H_{n,n+1} &= \frac{C(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t} + \eta_{n,n+1}\left[\xi K(t) - E_{n,n+1}(C(t))\right] \\ &\quad +\zeta^{n+1}_{n,n+1}(\lambda^{n+1}L - C(t)) + \zeta^{n}_{n,n+1}(C(t) - \lambda^n L) \end{aligned} \tag{12}$$

where $\eta_{n,n+1}$ is the co-state variable, $\zeta^{n+1}_{n,n+1}$ and $\zeta^{n}_{n,n+1}$ are the Lagrangian multipliers for the two constraints $\lambda^{n+1}L - C(t) \geq 0$ and $C(t) - \lambda^n L \geq 0$, respectively. The first order and K-T conditions are

$$\frac{\partial H_{n,n+1}}{\partial C} = C(t)^{-\sigma}e^{-\rho t} - \eta_{n,n+1}\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n} - \zeta^{n+1}_{n,n+1} + \zeta^{n}_{n,n+1} = 0, \tag{13}$$

$$\begin{aligned} \zeta^{n+1}_{n,n+1}(\lambda^{n+1}L - C(t)) = 0,\ \zeta^{n+1}_{n,n+1} \geq 0, \lambda^{n+1}L - C(t) \geq 0 \\ \zeta^{n}_{n,n+1}(C(t) - \lambda^n L) = 0,\ \zeta^{n}_{n,n+1} \geq 0, C(t) - \lambda^n L \geq 0. \end{aligned}$$

We also have

$$\eta'_{n,n+1}(t) = -\frac{\partial H_{n,n+1}}{\partial K} = -\eta_{n,n+1}\xi. \tag{14}$$

In particular, when $C(t) \in (\lambda^n L, \lambda^{n+1}L)$, $\zeta^{n+1}_{n,n+1} = \zeta^{n}_{n,n+1} = 0$, and equation (13) becomes

$$C(t)^{-\sigma}e^{-\rho t} = \eta_{n,n+1}\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}. \tag{15}$$

The left hand side is the marginal utility gain by increasing one unit of aggregate consumption, while the right hand side is the marginal utility loss due to the decrease in capital because of that additional unit of consumption, which by chain's rule can be decomposed into two multiplicative terms: The marginal utility of capital $\eta_{n,n+1}$ and the marginal capital requirement for each additional unit of aggregate consumption $\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}$ (see Table 1). Taking log of both sides of equation (15) and differentiating with respect to t, we have:

$$\frac{\dot{C}(t)}{C(t)} = \frac{\xi - \rho}{\sigma} \tag{16}$$

for $t_n \leq t \leq t_{n+1}$ for any $n \geq 0$. The strictly concave utility function implies that the optimal consumption flow $C(t)$ must be continuous and sufficiently smooth (with no kinks) throughout the time, hence from (16) we obtain:

$$C(t) = C(t_0)e^{\frac{\xi-\rho}{\sigma}(t-t_0)} \text{ for any } t \geq t_0. \tag{17}$$

Following Kamien and Schwartz (1991), we have two additional necessary conditions at $t = t_{n+1}$:

$$H_{n,n+1}(t_{n+1}) = H_{n+1,n+2}(t_{n+1}) \tag{18}$$

$$\eta_{n,n+1}(t_{n+1}) = \eta_{n+1,n+2}(t_{n+1}) \tag{19}$$

Substituting equations (18) and (19) into (12), we can verify that $K^-(t_{n+1}) = K^+(t_{n+1})$. In other words, $K(t)$ is indeed continuous.

Observe that $C(t_0)e^{\frac{\xi-\rho}{\sigma}(t_n-t_0)} = C(t_n) = \lambda^n L$, so $t_n = \frac{\log\frac{\lambda^n L}{C(t_0)} + \frac{\xi-\rho}{\sigma}t_0}{\frac{\xi-\rho}{\sigma}}$. Define $m_n \equiv t_{n+1} - t_n$, which measures the length of the period when both good n and $n+1$ are produced. It is easy to see that

$$m_n = m \equiv \frac{\sigma \log \lambda}{\xi - \rho}, \forall n \geq 1 \tag{20}$$

This result is summarized in the following proposition, where the goods at different levels should be interpreted as different industries.[9] These industries differ in the capital intensities of their production technologies.

Proposition 2 *The duration of each diversification cone for goods n and $n+1$is identical for all $n \geq 1$. The speed of industrial upgrading (measured by $\frac{1}{m}$) strictly increases with the efficiency of the production of capital goods, ξ, and intertemporal elasticity of substitution, $\frac{1}{\sigma}$, but strictly decreases with the quality gap λ and time discount rate ρ.*

The intuition is the following: when the household is more impatient (larger ρ), it will consume more and save less, causing the industrial upgrade to slow down. When the quality gap is larger (larger λ), the marginal utility of the current goods are bigger, therefore it pays to stay longer. When the production of the capital good becomes more efficient (ξ), capital can be accumulated faster, so the upgrade speed is increased. When the aggregate consumption is more substitutable across time, the household is more willing to sacrifice the current consumption so long as the aggregate consumption in the future can get sufficiently larger, which requires quicker industrial upgrading.

4 Industrial Dynamics

We are now ready to derive the industrial dynamics for the entire time period. The industrial dynamics depends on the initial capital stock, $K(0)$. We show in the

[9] To obtain closed-form solutions for the whole dynamics with infinite dimensional commodity space, we make the strong assumption that different industrial goods are perfectly substitutable so that some industries can die out. However, we believe this assumption is not crucial for our main result in industrial dynamics, depending on how the model is interpreted. For example, we may alternatively interpret each good n as a composite of several imperfectly substitutable industrial goods (such as food, clothes, electronics, etc.) similar to Acemoglu and Guerrieri (2008). The quality of the goods in each of these industries will improve over time, and the capital intensity of their technologies will also increase over time, which is reflected in the properties of the aggregate good n. We conjecture that the aggregate output of each industry (weighted sum over all the goods of different qualities in the same industry) will maintain the inverse-V-shaped dynamic pattern, although no industries will vanish (but some goods at certain quality levels will). We will leave this for future research.

Appendix that there exists a series of increasing constants, $\vartheta_0, \vartheta_1, \cdots, \vartheta_n, \vartheta_{n+1}, \cdots$, such that if $0 < K(0) \leq \vartheta_0$, the economy will start by producing good 0 only until the capital stock reaches ϑ_0 (Appendix 3 fully characterizes this case); if $\vartheta_0 < K(0) \leq \vartheta_1$, the economy will start by producing goods 0 and 1; if $\vartheta_n < K(0) \leq \vartheta_{n+1}$, the economy will start by producing goods n and $n+1$. Furthermore, we can show that $K(t_n) \equiv \vartheta_n$ for any $K(0) < \vartheta_n$. That is, irrespective of the level of initial capital stock, the economy always starts to produce good $n+1$ when its capital stock reaches ϑ_n.

To be more concrete, let us consider the case when $\vartheta_0 < K(0) \leq \vartheta_1$, where threshold values ϑ_0 and ϑ_1 can be explicitly solved out (see the Appendix). That is, the economy will start by producing goods 0 and 1. Using equation (17) and Table 1, we know that when $t \in [0, t_1]$,

$$E(t) = \frac{a}{\lambda - 1}(C(t) - L) = \frac{a}{\lambda - 1}(C(0)e^{\frac{\xi-\rho}{\sigma}t} - L).$$

Correspondingly,

$$\dot{K} = \xi K(t) - \frac{a}{\lambda - 1}(C(0)e^{\frac{\xi-\rho}{\sigma}t} - L).$$

Solving this first-order differential equation with the condition $K(0) = K_0$, we obtain

$$K(t) = \frac{-\frac{aC(0)}{\lambda-1}}{\frac{\xi-\rho}{\sigma} - \xi}e^{\frac{\xi-\rho}{\sigma}t} + \frac{-aL}{\xi(\lambda - 1)} + \left[K_0 + \frac{\frac{aC(0)}{\lambda-1}}{\frac{\xi-\rho}{\sigma} - \xi} + \frac{aL}{\xi(\lambda - 1)}\right]e^{\xi t},$$

which yields

$$\vartheta_1 \equiv K(t_1) = \frac{-\frac{a\lambda L}{\lambda-1}}{\frac{\xi-\rho}{\sigma} - \xi} + \frac{-aL}{\xi(\lambda - 1)} + \left[K_0 + \frac{\frac{aC(0)}{\lambda-1}}{\frac{\xi-\rho}{\sigma} - \xi} + \frac{aL}{\xi(\lambda - 1)}\right]\left(\frac{\lambda L}{C(0)}\right)^{\frac{\xi\sigma}{\xi-\rho}}. \quad (21)$$

When $t \in [t_n, t_{n+1}]$, for any $n \geq 1$, the transition equation of capital stock (10) becomes

$$\dot{K} = \xi K(t) - \left[C(0) e^{\frac{\xi-\rho}{\sigma}t} - \frac{\lambda^n (a-\lambda)}{a-1} L \right] \frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n} \text{ when } t \in [t_n, t_{n+1}], \text{ for any } n \geq 1. \tag{22}$$

Solving the differential equation (22), we obtain:

$$K(t) = \alpha_n + \beta_n e^{\frac{\xi-\rho}{\sigma}t} + \gamma_n e^{\xi t} \text{ when } t \in [t_n, t_{n+1}], \text{ for any } n \geq 1 \tag{23}$$

where

$$\begin{aligned}
\alpha_n &= -\left(\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\right) \frac{\lambda^n (a-\lambda) L}{\xi (a-1)}, \\
\beta_n &= -\left(\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\right) \frac{C(0)}{\left(\frac{\xi-\rho}{\sigma} - \xi\right)}, \\
\gamma_n &= \left[\frac{\lambda^n L}{C(0)}\right]^{\frac{-\xi\sigma}{\xi-\rho}} \left\{ \vartheta_n + \frac{\left(a^{n+1}-a^n\right) L}{\lambda - 1} \left[\frac{1}{\left(\frac{\xi-\rho}{\sigma} - \xi\right)} + \frac{(a-\lambda)}{\xi (a-1)} \right] \right\}.
\end{aligned}$$

Note that $C(0)$ can be uniquely determined by using the transversality condition (see the Appendix). $\{\vartheta_n\}_{n=2}^{\infty}$ are all constants, which can sequentially pinned down: $\vartheta_n \equiv K(t_n)$ can be computed from equation (23) with $K(t_{n-1})$ known.

For each individual industry, using equation (17) and Table 1, we have

$$c_n^*(t) = \begin{cases} \frac{C(0)e^{\frac{\xi-\rho}{\sigma}t}}{\lambda^n - \lambda^{n-1}} - \frac{L}{\lambda - 1} & \text{when } t \in [t_{n-1}, t_n] \\ -\frac{C(0)e^{\frac{\xi-\rho}{\sigma}t}}{\lambda^{n+1} - \lambda^n} + \frac{\lambda L}{\lambda - 1}, & \text{when } t \in [t_n, t_{n+1}] \\ 0, & \text{otherwise} \end{cases}, \text{ for all } n \geq 2$$

$$c_1^*(t) = \begin{cases} \frac{C(0)e^{\frac{\xi-\rho}{\sigma}t} - L}{\lambda - 1}, & \text{when } t \in [t_0, t_1] \\ -\frac{C(0)e^{\frac{\xi-\rho}{\sigma}t}}{\lambda^2 - \lambda} + \frac{\lambda L}{\lambda - 1}, & \text{when } t \in [t_1, t_2] \\ 0, & \text{otherwise} \end{cases},$$

$$c_0^*(t) = \begin{cases} L - \frac{C(0)e^{\frac{\xi-\rho}{\sigma}t} - L}{\lambda - 1}, & \text{when } t \in [t_0, t_1] \\ 0, & \text{otherwise} \end{cases}.$$

This can be illustrated in the following inverse-V-shaped time pattern of industrial dynamics:

[Figure 5]

The above mathematical results can be read as follows:

Proposition 3 *There exist a strictly increasing and non-negative sequence of threshold values for capital stock,* $\{\vartheta_i\}_{i=0}^{\infty}$ *, which are all independent of the initial capital stock, such that the economy starts to produce good* n *when its capital stock* $K(t)$ *reaches* ϑ_{n-1}*.* $K(t)$ *evolves following the equation (23), while the total consumption* $C(t)$ *remains constant at* L *until* t_0*, after which it grows exponentially at the constant rate* $\frac{\xi-\rho}{\sigma}$*. The output of each industry evolves in an inverse-V-shaped pattern: When capital stock* $K(t)$ *reaches* ϑ_{n-1}*, good* n *enters the market and its output grows approximately at the constant rate* $\frac{\xi-\rho}{\sigma}$ *until capital stock* $K(t)$ *reaches* ϑ_n*; its output then declines approximately at the constant rate* $\frac{\xi-\rho}{\sigma}$*, and exits from the market at the time when* $K(t)$ *reaches* ϑ_{n+1}*.*[10]

5 "Mistakes" in Product Selection and Consequences

Our previous analysis characterizes the first-best industrial dynamics in a frictionless economy. The evolution of optimal production structure is determined by the change of capital per capita, $K(t)/L$, which evolves following equation (23). In this section, we will briefly discuss the welfare consequences if the goods chosen to produce are not the first-best ones. These “selection mistakes” may happen in real life for a variety of reasons. For example, the government in a developing economy may, for some political reasons, pursue a catching-up development strategy by prematurely pushing the economy to produce what is produced in the much more developed economies, which is too capital intensive (see Lin 2003, 2008), that is, the economy

[10] More rigorously, the sum of the good n's output and a constant, $c_n^*(t) + \frac{L}{\lambda-1}$, grows at the constant rate $\frac{\xi-\rho}{\sigma}$, and then $c_n^*(t) - \frac{\lambda L}{\lambda-1}$ declines at the constant rate $\frac{\xi-\rho}{\sigma}$.

is urged to produce good $n+s$ when good n is the first best for some $s \geq 1$. Or the market itself may fail to choose optimal products, due to incomplete information or firms' monopoly behavior, etc. Due to space limitations, we will focus our analysis on the ex-post welfare consequences per se while remaining agnostic about what causes those deviations from the first-best industrial selections.

[Figure 6]

The optimal evolution of capital stock $K(t)$, and capital expenditure $E(t)$ are represented by curve $KK(t)$ and $EE(t)$ in Figure 6, respectively. At time t_n, $K(t_n) = \vartheta_n$ and $E(t_n) = a^n L$. It is optimal for the economy to start producing good $n+1$ so that both good n and good $n+1$ should be produced at time interval (t_n, t_{n+1}). Now suppose, for some unexpected and exogenous reasons, the economy starts to produce goods $n+s$ and $n+s+1$ during the period $[t_n, t_{n+1}]$ for some $s > 0$. s measures the magnitude of the industrial deviation. So at time t_n, instead of using $E(t_n) = a^n L$, now suppose the economy chooses the expenditure $E(t_n) = a^{n+s} L$ and maintains the same consumption growth rate $\frac{\xi-\rho}{\sigma}$ during this period. We use the superscript "M" to denote all the variables after the economy is unexpectedly hit by this "mistake." What will the welfare consequence be?

Obviously at time t_n, the aggregate consumption jumps from $C(t_n)$ to $C^M(t_n)$. Capital expenditure $E^M(t)$ now evolves as follows:

$$E^M(t) = \left[C^M(t_n) e^{\frac{\xi-\rho}{\sigma}(t-t_n)} - \frac{\lambda^{n+s}(a-\lambda)}{a-1} L \right] \frac{a^{n+s+1} - a^{n+s}}{\lambda^{n+s+1} - \lambda^{n+s}}$$

for any $t \in [t_n, t_{n+1}]$. It is represented by curve $E_n^M E_{n+1}^M$ in Figure 6. Different from equation (22), the transition equation of capital now becomes

$$\dot{K}^M = \xi K^M(t) - \left[C^M(t_n) e^{\frac{\xi-\rho}{\sigma}(t-t_n)} - \frac{\lambda^{n+s}(a-\lambda)}{a-1} L \right] \frac{a^{n+s+1} - a^{n+s}}{\lambda^{n+s+1} - \lambda^{n+s}}, \text{ for any } t \in [t_n, t_{n+1}]. \tag{24}$$

If s is sufficiently large, the capital expenditure $E^M(t)$ exceeds $\xi K^M(t)$ and therefore $K^M(t)$ declines in time period $[t_n, t_{n+1}]$. In that case, capital stock evolves along the curve $K_n^M K_{n+1}^M$ which is not only below the first-best path $KK(t)$ but also decreasing. At time t_{n+1}, $K^M(t_{n+1}) < \vartheta_{n+1}$. Let us assume $\vartheta_h \leq K^M(t_{n+1}) \leq \vartheta_{h+1}$ for some $0 < h < n$. Suppose at time $t = t_{n+1}$ the government (or agents) in the economy suddenly realizes that it made mistakes during the period $[t_n, t_{n+1}]$. What should it do? There are two scenarios:

In Scenario 1, let us assume there is no adjustment cost to rectify the mistake. The economy can freely re-optimize everything at $t = t_{n+1}$ at given capital stock $K = K^M(t_{n+1})$. Since $\vartheta_h \leq K^M(t_{n+1}) \leq \vartheta_{h+1}$, the economy will immediately downgrade its production structure and start producing goods h and $h + 1$. The corresponding capital expenditure $E^M(t_{n+1}^+)$ must be between $a^h L$ and $a^{h+1} L$, and the economy will then follow the optimal path as if it began with $K = K^M(t_{n+1})$. The evolution paths for K^M and E^M are represented by $KK_n^M K_{n+1}^M K_1^M(t)$ and $EAE_n^M E_{n+1}^M BE_1^M(t)$, respectively. Consumers enjoy the goods with qualities higher than the optimal, $n + s$ and $n + s + 1$, in time period $[t_n, t_{n+1}]$, but have to adjust the economy at a much lower level thereafter. The consumer's life utility following the mistaken path $KK_n^M K_{n+1}^M K_1^M(t)$ certainly is lower than the optimum and can be computed. Similarly, if the country saves more capital than the optimum, and produces goods $n - s$ and $n - s + 1$ in $[t_n, t_{n+1}]$, the country will re-optimize and start to produce goods j and $j + 1$ for some $j \geq n + 1$ after $t = t_{n+1}$; that also lowers consumer's lifetime total utility.

In Scenario 2, let us assume that the production structure can not be reversed and maintain the full employment assumption. That is, when the economy starts to produce good m, it can not produce any good with the quality lower than m in the future. Full employment implies that the quantity of total consumption can not decrease either. The industrial irreversibility may be due to the fact that the physical capital used to produce high quality goods can not be reversed to produce lower quality goods, or consumers who are used to consuming high-quality goods are simply addicted to their consumer behavior, or political groups in current industries may lobby against the structural adjustment. Under this assumption, the optimal choice is to produce the lowest quality good constrained by the downward rigidity constraint $n \geq h + s + 1$. Therefore, at each time point after $t = t_{n+1}$ capital expenditure E must always equal $a^{h+s+1}L$. As too little capital is saved, $K^M(t)$ continues to decline after t_{n+1} and will be exhausted at time T. The evolution paths for K^M and E^M in Scenario 2 are represented by $KK_n^M K_{n+1}^M K_2^M(t)$ and $EAE_n^M E_{n+1}^M E_2^M(t)T$, respectively. Due to the adjustment rigidity, a short-term mistake in production selection hurts the economy permanently by fully exhausting all capital stock. The economy falls into a poverty trap without any growth and can only afford to produce good 0 forever after time T, if that is allowed after T.

Scenarios 1 and 2 provide two extreme cases for the effect of mistakes in production selection. Without adjustment cost, one period mistake to produce products above the optimum lowers social welfare and postpones future product upgrading. When the production structure is not reversible, however, one period over-expenditure in capital may permanently degenerate the economy and destroy the hope for growth. We can impose the same downward adjustment rigidity constraint on the economy analyzed in the previous two sections, but nothing changes because this adjustment constraint is simply not binding.

An intermediate adjustment cost function between Scenarios 1 and 2 may be more realistic, but also more complicated to analyze. For example, we may assume that right after time t_{n+1}, capital stock $K(t_{n+1}^+) = \Phi([h - (n+1)])K(t_{n+1})$ where $\Phi(.)$ is the adjustment cost function. $\Phi(0) = 1$, $0 \leq \Phi(.) \leq 1$, and $\Phi(.)$ becomes larger if the absolute value of $[h - (n+1)]$ increases. If the economy maintains its current production structures, there will be no adjustment cost. Otherwise, a more radical adjustment incurs a higher adjustment cost. Scenario 1 represents the case that $\Phi(.) \equiv 1$. For Scenario 2, it simply refers to the case that $\Phi(.) \equiv 0$ if

$h-(n+1) < 0$. When $\Phi(.)$ is intermediate, the economy may not immediately adjust her production structures to the optimal products h and $h+1$. Instead, the country may gradually downgrade its production structure to the optimal one determined by its endowment structure. A thorough formal exploration is beyond the scope of this paper, but studies of such optimal adjustment in production structures seem very interesting, although quite limited in the literature.

In the real world, we observed successful examples of industrial upgrading in Japan, South Korea, and Taiwan from the 1950s to the 1980s. That may be partly attributed to the fact that the government in those economies pursued the right industrial development policies that were consistent with their endowment structures. However, there are many examples of less successful development with "non-optimal" industrial dynamics in developing economies including China, India, Russia, and many other Eastern European and Latin American countries during various historical periods. These countries all erroneously pursued development strategies that defied their comparative advantages by naively trying to quickly mimic the production structure of the developed economies and, hence, prematurely built too many heavy industries that were inconsistent with the economy's endowment structure. That ultimately resulted in serious economic stagnation and caused huge welfare loss. Just as predicted by our above analysis, the welfare consequence is even worse when the industrial adjustment is more costly. Indeed, we frequently observe prolonged difficult periods of adjustment in many economies undertaking reforms. For instance, it has been more than 15 years since the regime switched in Russia and the country's endowment structure also dramatically deteriorated. Nevertheless, military and some other heavy industries are still the main supporting industries in that country (Lin 2003, 2009).

Serious welfare consequences may result from the government's failure to recognize that the optimal industries are actually endogenous to the development stage (capital endowment). Such important policy implications, although indirectly derived from this model, may not be obvious from the standard one-sector or multi-sector growth models.

6 Conclusion

We have developed a tractable infinite-horizon general-equilibrium model to analyze the optimal industrial structure and its dynamics in a closed developing economy. Explicit solutions are obtained to fully characterize the whole economic dynamics (including the transitional dynamics), although the commodity space is infinitely dimensional. Our model generates an inverse-V-shaped dynamic pattern of industrial change, which is widely observed in the real world but rarely, if ever, formalized in growth models. The engine that drives this continuous structural change is the increasing relative abundance of capital in the endowment structure. In a closed economy, capital keeps accumulating because the capital goods are produced with Arrow's AK model technology, which can be interpreted as learning-by-doing in the capital production. This endogenous technological change in the capital good production gives us the sustained and constant economic growth in the aggregate good consumption. However, since the industries are upgrading in a waxing-and-waning fashion all the time, the growth rates of each individual industrial good are shown to be changing along the process. We highlight the endogeneity of the industrial structures and its dynamics: The optimal industrial choice and industrial dynamics are dictated by the economy's endowment structure and its change.

Economic growth and industrial upgrading are two crucial and integrated aspects of sustained economic development. On the quantitative side, sustained economic development requires the sustainable growth of per capita income; on the structural side, sustainable economic development typically entails the continuous upgrading and transformation of industries. Most existing growth models postulate the same aggregate production function (with changing inputs and productivity) for all the countries at different development stages, and, thus, naturally focus on the quantitative side of economic aggregates, which have been guiding economists to conduct many insightful policy studies such as how to boost human and physical capital accumulation and how to enhance technological improvement, and so forth. However, the structural side is largely ignored by these models. This perhaps accounts for the more serious shortage of academic and policy research related to structural changes in development: How to help the economy identify the optimal products and industries to develop at each different development stage, which kind of financial institutions can best serve the corresponding industrial structures at different stages, how to facilitate the structural transformation in the process of labor and capital reallocation across industries, how openness may affect a country's industry upgrading, and, consequently, what the optimal industrial, financial, trade, and many other macroeconomic policies should be at different development stages, so on and so forth. We hope the model developed in this paper may serve as a useful starting point to address all these fundamental issues.

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7 Appendix

7.1 Appendix 1

In Appendix 1, we show that in the competitive equilibrium, if two different industrial goods are produced simultaneously, these two goods have to be adjacent in quality.

Proof: By contradiction. Suppose the above statement is not true, then there can exist some good j and good m such that $1 \leq j \leq m-2$, and c_j and c_m are both strictly positive in the equilibrium. Then

$$MRS_{m,j} = \lambda^{m-j} = \frac{a^m r + w}{a^j r + w},$$

which means

$$\frac{r}{w} = \frac{\lambda^{m-j} - 1}{a^j (a^{m-j} - \lambda^{m-j})}.$$

Now we show that at this relative price, good $j+1$ strictly dominates good j, that is

$$MRS_{j+1,j} - \frac{a^{j+1} r + w}{a^j r + w} = \lambda - \frac{a^{j+1} \frac{\lambda^{m-j}-1}{a^j (a^{m-j} - \lambda^{m-j})} + 1}{a^j \frac{\lambda^{m-j}-1}{a^j (a^{m-j} - \lambda^{m-j})} + 1} > 0,$$

which is equivalent to

$$\frac{\lambda^n - 1}{a^n - \lambda^n} < \frac{1-\lambda}{\lambda - a}, \text{ for some integer } n \equiv m - j \geq 2. \tag{25}$$

This is true because

$$\begin{aligned}\frac{\lambda^n - 1}{a^n - \lambda^n} &= \frac{(\lambda - 1)\sum_{i=0}^{n-1} \lambda^i}{\lambda^n \left(\left(\frac{a}{\lambda}\right)^n - 1\right)} = \frac{(\lambda - 1)\sum_{i=0}^{n-1} \lambda^i}{\lambda^n \left(\left(\frac{a}{\lambda}\right) - 1\right)\sum_{i=0}^{n-1} \left(\frac{a}{\lambda}\right)^i} \\ &= \frac{(\lambda - 1)\sum_{i=0}^{n-1} \lambda^i}{\lambda^{n-1} (a - \lambda)\sum_{i=0}^{n-1} \left(\frac{a}{\lambda}\right)^i} = \frac{(\lambda - 1)\sum_{i=0}^{n-1} \lambda^i}{(a - \lambda)\sum_{i=0}^{n-1} \lambda^i a^{n-1-i}} < \frac{\lambda - 1}{a - \lambda} \text{for any } n \geq 2.\end{aligned}$$

Therefore, it contradicts that good j is produced and consumed.

It's straightforward to show that it is possible to have both good 0 and good 1 under some conditions.

Now we need to show that if some good $n \geq 2$ is the only industrial good that's produced, then good 0 can not be produced. From (6), we know that the household would strictly prefer good n to good $n+1$ if

$$\frac{\lambda_{n+1}}{\lambda_n} < \frac{p_{n+1}}{p_n} = \frac{w + a_{n+1} r}{w + a_n r}, \text{for } n = 1, ..$$

or equivalently,

$$\frac{r}{w} > \frac{\lambda - 1}{a^{n+1} - \lambda a^n}, \tag{26}$$

and good n is strictly preferred to good $n-1$ if and only if

$$\frac{r}{w} < \frac{\lambda - 1}{a^n - \lambda a^{n-1}}. \tag{27}$$

This means

$$\frac{\lambda - 1}{a^{n+1} - \lambda a^n} < \frac{r}{w} < \frac{\lambda - 1}{a^n - \lambda a^{n-1}}. \tag{28}$$

If $c_0 > 0$, then $\lambda_n = \frac{w + a_n r}{w}$ because of (6), that is

$$\frac{\lambda^n - 1}{a^n} = \frac{r}{w}.$$

So we must have

$$\frac{\lambda - 1}{a^{n+1} - \lambda a^n} < \frac{\lambda^n - 1}{a^n} < \frac{\lambda - 1}{a^n - \lambda a^{n-1}},$$

where the second inequality is equivalent to

$$\frac{\lambda^n - 1}{\lambda - 1} < \frac{a}{a - \lambda}.$$

However, since $a - 1 > \lambda$, there will exist no integer $n \geq 2$ that can satisfy the above inequality because the left hand side is no smaller than $\lambda + 1$. This implies that $c_0 = 0$.

7.2 Appendix 2

In Appendix 2, we solve for the initial value of total consumption $C(0)$ when $\vartheta_0 < K(0) \leq \vartheta_1$, and also show how to derive the threshold values for $\vartheta_i, i = 0, 1, 2, ...$ The transversality condition is derived from

$$\lim_{t \to \infty} H(t) = 0,$$

so

$$\lim_{t \to \infty} \left[\frac{C(t)^{1-\sigma} - 1}{1 - \sigma} e^{-\rho t} + \eta_{n(t),n(t)+1} \left[\xi K(t) - E_{n(t),n(t)+1}(C(t)) \right] \right] = 0.$$

Note that

$$\begin{aligned}
&\lim_{t\to\infty}\left[\frac{C(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t}+\eta_{n(t),n(t)+1}\left[\xi K(t)-E_{n(t),n(t)+1}(C(t))\right]\right]\\
&=\lim_{t\to\infty}\left[\frac{C(0)^{1-\sigma}e^{\frac{\xi-\rho}{\sigma}(1-\sigma)t}}{1-\sigma}e^{-\rho t}+\eta_{n(t),n(t)+1}\left[\xi K(t)-E_{n(t),n(t)+1}(C(t))\right]\right]\\
&=\lim_{t\to\infty}\left[\eta_{n(t),n(t)+1}\left[\xi K(t)-E_{n(t),n(t)+1}(C(t))\right]\right]\\
&=\lim_{t\to\infty}\left\{\eta_{(0)}e^{-\xi t}\left[\xi K(t)-\left[C(0)e^{\frac{\xi-\rho}{\sigma}t}-\frac{\lambda^{n(t)}(a-\lambda)}{a-1}L\right]\frac{a^{n(t)+1}-a^{n(t)}}{\lambda^{n(t)+1}-\lambda^{n(t)}}\right]\right\}\\
&=\lim_{t\to\infty}\left\{\eta_{(0)}\left[\xi K(t)e^{-\xi t}-\left[-\frac{e^{-\xi t}(a-\lambda)}{a-1}L\right]\frac{a^{n(t)+1}-a^{n(t)}}{\lambda-1}\right]\right\}\\
&=\lim_{t\to\infty}K(t)e^{-\xi t},
\end{aligned}$$

thus we must have $\lim_{t\to\infty}K(t)e^{-\xi t}=0$. When $t\in[t_0=0,t_1]$,

$$E(t)=\frac{a}{\lambda-1}(C(t)-L)=\frac{a}{\lambda-1}(C(0)e^{\frac{\xi-\rho}{\sigma}t}-L),$$

Correspondingly,

$$\dot{K}=\xi K(t)-E(C(t))=\xi K(t)-\frac{a}{\lambda-1}(C(0)e^{\frac{\xi-\rho}{\sigma}t}-L)$$

Solving this first-order differential equation with the condition $K(0)=K_0$, we obtain

$$K(t)=\frac{-\frac{aC(0)}{\lambda-1}}{\frac{\xi-\rho}{\sigma}-\xi}e^{\frac{\xi-\rho}{\sigma}t}+\frac{-aL}{\xi\left(\lambda-1\right)}+\left[K_0+\frac{\frac{aC(0)}{\lambda-1}}{\frac{\xi-\rho}{\sigma}-\xi}+\frac{aL}{\xi\left(\lambda-1\right)}\right]e^{\xi t},\tag{29}$$

from which we obtain

$$K(t_1)=\frac{-\frac{a\lambda L}{\lambda-1}}{\frac{\xi-\rho}{\sigma}-\xi}+\frac{-aL}{\xi\left(\lambda-1\right)}+\left[K_0+\frac{\frac{aC(0)}{\lambda-1}}{\frac{\xi-\rho}{\sigma}-\xi}+\frac{aL}{\xi\left(\lambda-1\right)}\right]\left(\frac{\lambda L}{C(0)}\right)^{\frac{\xi\sigma}{\xi-\rho}}.\tag{30}$$

When $t\in[t_n,t_{n+1}]$ for $\forall n\geq 1$, we have

$$K(t)=-\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\left[\frac{C(0)e^{\frac{\xi-\rho}{\sigma}t}}{\left(\frac{\xi-\rho}{\sigma}-\xi\right)}+\frac{\lambda^n(a-\lambda)L}{\xi\left(a-1\right)}\right]+\theta_{n.n+1}e^{\xi t}.\tag{31}$$

which gives

$$\theta_{n.n+1}=\left[\frac{\lambda^nL}{C(0)}\right]^{\frac{-\xi\sigma}{\xi-\rho}}\left\{K(t_n)+\frac{a^{n+1}-a^n}{\lambda-1}L\left[\frac{1}{\left(\frac{\xi-\rho}{\sigma}-\xi\right)}+\frac{(a-\lambda)}{\xi\left(a-1\right)}\right]\right\}\tag{32}$$

Therefore, we obtain equation (23). Substituting $t = t_{n+1} = \frac{\log \frac{\lambda^{n+1}L}{C(0)}}{\frac{\xi-\rho}{\sigma}}$ into (23), we obtain

$$K(t_{n+1}) = \lambda^{\frac{\sigma\xi}{\xi-\rho}} K(t_n) + \frac{a^{n+1} - a^n}{\lambda - 1} L \left[\frac{\lambda^{\frac{\sigma\xi}{\xi-\rho}} - \lambda}{\left(\frac{\xi-\rho}{\sigma} - \xi\right)} + \frac{(a-\lambda)(\lambda^{\frac{\sigma\xi}{\xi-\rho}} - 1)}{\xi\,(a-1)} \right]$$

Using the recursive induction, we get

$$K(t_n) = \lambda^{\frac{(n-1)\sigma\xi}{\xi-\rho}} K(t_1) + (a-1)B\lambda^{\frac{(n-2)\sigma\xi}{\xi-\rho}} \frac{a\left[1 - \left(a\lambda^{\frac{-\sigma\xi}{\xi-\rho}}\right)^{n-1}\right]}{1 - a\lambda^{\frac{-\sigma\xi}{\xi-\rho}}}, \text{ for any } n \geq 2 \quad (33)$$

where B is a parameter defined as

$$B \equiv \frac{L}{\lambda - 1} \left[\frac{\lambda^{\frac{\sigma\xi}{\xi-\rho}} - \lambda}{\frac{\xi-\rho}{\sigma} - \xi} + \frac{(a-\lambda)\left(\lambda^{\frac{\sigma\xi}{\xi-\rho}} - 1\right)}{\xi(a-1)} \right].$$

The transversality condition $\lim_{t\to\infty} K(t)e^{-\xi t} = 0$ implies that

$$\lambda^{\frac{-\sigma\xi}{\xi-\rho}} K(t_1) + (a-1)B\lambda^{\frac{-2\sigma\xi}{\xi-\rho}} \frac{a}{1 - a\lambda^{\frac{-\sigma\xi}{\xi-\rho}}} = 0,$$

so

$$K(t_1) = -\frac{(a-1)B\lambda^{\frac{-\sigma\xi}{\xi-\rho}} a}{1 - a\lambda^{\frac{-\sigma\xi}{\xi-\rho}}}. \quad (34)$$

To make the analysis rigorous and relevant, we will have impose two additional assumptions.

$$a < \lambda^{\frac{\sigma\xi}{\xi-\rho}}, \quad (35)$$

and

$$\xi > \frac{\rho(a-\lambda)\left(\lambda^{\frac{\sigma\xi}{\xi-\rho}} - 1\right)}{\sigma\left(a - \lambda^{\frac{\sigma\xi}{\xi-\rho}}\right)(1-\lambda) + (a-\lambda)\left(\lambda^{\frac{\sigma\xi}{\xi-\rho}} - 1\right)}. \quad (36)$$

(35) is needed to ensure $\vartheta_0 > 0$. It says that capital accumulation speed ξ has to be sufficiently large relative to the capital intensity parameter a so that industrial upgrading is indeed happening. (36) ensures $\vartheta_1 \equiv K(t_1) > 0$ and $B < 0$. Note that (36) guarantees that $\xi > \rho$.

According to (30), we have

$$\left[K_0+\frac{aC(0)}{(\lambda-1)\left(\frac{\xi-\rho}{\sigma}-\xi\right)}+\frac{aL}{\xi(\lambda-1)}\right]\left(\frac{\lambda L}{C(0)}\right)^{\frac{\xi\sigma}{\xi-\rho}}$$
$$=\quad\frac{a\lambda L}{(\lambda-1)\left(\frac{\xi-\rho}{\sigma}-\xi\right)}+\frac{aL}{\xi(\lambda-1)}-\frac{(a-1)B\lambda^{\frac{-\sigma\xi}{\xi-\rho}}a}{1-a\lambda^{\frac{-\sigma\xi}{\xi-\rho}}}. \tag{37}$$

We can verify that the right hand side is strictly positive. Observe that the left hand side is a strictly decreasing function of $C(0)$, therefore we can uniquely pin down the optimal $C^*(0)$. (37) immediately implies $\frac{\partial C^*(0)}{\partial K_0}>0$ and $\frac{\partial C^*(0)}{\partial L}>0$.

Note that (34) implies that $K(t_1)$ does not depend on $K(0)$, therefore (33) tells that $K(t_n)$ for all $n\geq 1$ are independent from $K(0)$.

To ensure $C^*(0)\leq\lambda L$, we need $K_0\leq-\frac{(a-1)B\lambda^{\frac{-\sigma\xi}{\xi-\rho}}a}{1-a\lambda^{\frac{-\sigma\xi}{\xi-\rho}}}$, which requires

$$K_0\leq\vartheta_1\equiv-\frac{\lambda^{\frac{-\sigma\xi}{\xi-\rho}}a}{1-a\lambda^{\frac{-\sigma\xi}{\xi-\rho}}}\frac{1}{\lambda-1}\left[\frac{\xi\left(a-\lambda^{\frac{\sigma\xi}{\xi-\rho}}\right)(1-\lambda)+\frac{\xi-\rho}{\sigma}(a-\lambda)\left(\lambda^{\frac{\sigma\xi}{\xi-\rho}}-1\right)}{\left(\frac{\xi-\rho}{\sigma}-\xi\right)\xi}\right]L.$$

We also need to ensure $C^*(0)>L$, which requires

$$\left[K_0+\frac{aL}{(\lambda-1)\left(\frac{\xi-\rho}{\sigma}-\xi\right)}+\frac{aL}{\xi(\lambda-1)}\right]\lambda^{\frac{\xi\sigma}{\xi-\rho}}>\frac{a\lambda L}{(\lambda-1)\left(\frac{\xi-\rho}{\sigma}-\xi\right)}+\frac{aL}{\xi(\lambda-1)}-\frac{(a-1)B\lambda^{\frac{-\sigma\xi}{\xi-\rho}}a}{1-a\lambda^{\frac{-\sigma\xi}{\xi-\rho}}},$$

that is

$$K_0>\vartheta_0\equiv\frac{a}{(\lambda-1)}\frac{\frac{\xi-\rho}{\sigma}}{\left(\frac{\xi-\rho}{\sigma}-\xi\right)\xi}\left[\frac{\left(1-\lambda^{1-\frac{\sigma\xi}{\xi-\rho}}\right)(1-\lambda^{\frac{\xi\sigma}{\xi-\rho}})}{\lambda^{\frac{\xi\sigma}{\xi-\rho}}\left(1-a\lambda^{\frac{-\sigma\xi}{\xi-\rho}}\right)}\right]L.$$

7.3 Appendix 3

In Appendix 3, we prove that there exists a series of constant numbers, $\vartheta_0,\vartheta_1,\cdots,\vartheta_n,\vartheta_{n+1},\cdots$; if $0<K(0)\leq\vartheta_0$, the economy will start from producing good 0 only until the capital stock reaches ϑ_0; if $\vartheta_0<K(0)\leq\vartheta_1$, the economy will start from producing goods 0 and 1; if $\vartheta_n<K(0)\leq\vartheta_{n+1}$, the economy will start from producing goods n and $n+1$. Furthermore, $K(t_n)=\vartheta_n$ for any value of $K(0)<\vartheta_n$.

Now let us characterize the solution to the above dynamic problem when $K_0\in(0,\vartheta_0]$ while keeping all the other assumptions unchanged. The economy must start by producing good 0 only. The discounted-value Hamiltonian with the Lagrangian multipliers is the following

$$H_0 = \frac{C(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} + \eta_0 \xi K(t) + \zeta_0^0 (L - C(t)).$$

First order condition and K-T condition are

$$\begin{aligned} C(t)^{-\sigma} e^{-\rho t} &= \zeta_0^0; \\ \zeta_0^0 (L - C(t)) &= 0; \\ L - C(t) &= 0 \text{ when } \zeta_0^0 > 0. \end{aligned}$$

and

$$\eta_0 = -\frac{\partial H_0}{\partial K} = -\eta_0 \xi.$$

They immediately imply that $C^*(t) = L$. This is because labor entails no utility cost for the household therefore $C(t)$ must be equal to L when only good 0 is produced. No capital is used for production and therefore

$$\dot{K}(t) = \xi K(t).$$

When capital stock K exceeds ϑ_0 by an infinitessimal amount, the economy produces both good 0 and good 1. From that point on, the problem is exactly the same as the one we have just solved in the main text. Let t_0 denote the time point when K equals ϑ_0. Then

$$K_0 e^{\xi t_0} = \vartheta_0,$$

so $t_0 = \frac{\log \frac{\vartheta_0}{K_0}}{\xi}$. Therefore

$$C^*(t) = \begin{cases} L, & \text{when} \quad t \le t_0 \\ L e^{\frac{\xi-\rho}{\sigma}(t-t_0)}, & \text{when} \quad t > t_0 \end{cases}.$$

Let t_j denote the time point when only good j is produced, for any $j \ge 1$. Observe that $L e^{\frac{\xi-\rho}{\sigma}(t_j - t_0)} = C(t_j) = \lambda^j L$, so $t_j = t_0 + \left(\frac{\sigma \log \lambda}{\xi - \rho}\right) j$.

Correspondingly, the capital stock on the equilibrium path is given by

$$K(t) = \begin{cases} K_0 e^{\xi t}, & \text{for} \quad t \in [0, t_0] \\ \frac{-\frac{aL}{\lambda-1}}{\frac{\xi-\rho}{\sigma} - \xi} e^{\frac{\xi-\rho}{\sigma}(t-t_0)} + \frac{-aL}{\xi(\lambda-1)} + \left[\vartheta_0 + \frac{\frac{aL}{\lambda-1}}{\frac{\xi-\rho}{\sigma} - \xi} + \frac{aL}{\xi(\lambda-1)}\right] e^{\xi(t-t_0)}, & \text{for} \quad t \in [t_0, t_1] \\ F(t), & \text{for} \quad t \in [t_n, t_{n+1}], \text{ any } n \ge 1 \end{cases},$$

where

$$F(t) \equiv -\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\left[\frac{Le^{\frac{\xi-\rho}{\sigma}t}}{\left(\frac{\xi-\rho}{\sigma}-\xi\right)}+\frac{\lambda^n(a-\lambda)L}{\xi\left(a-1\right)}\right]$$
$$+\left[\lambda^n\right]^{\frac{-\xi\sigma}{\xi-\rho}}\left\{K(t_n)+\frac{a^{n+1}-a^n}{\lambda-1}L\left[\frac{1}{\left(\frac{\xi-\rho}{\sigma}-\xi\right)}+\frac{(a-\lambda)}{\xi\left(a-1\right)}\right]\right\}e^{\xi(t-t_0)},$$

$t_0=\frac{\log\frac{\vartheta_0}{K_0}}{\xi}$, $t_j=t_0+\left(\frac{\sigma\log\lambda}{\xi-\rho}\right)j$ for any $j\geq 1$, $K(t_0)=\vartheta_0$, and $K(t_n)$ is exactly the same as before for any $n\geq 1$.

Using the similar algorithm, we can fully specify the transitional dynamics when $K_0>\vartheta_1$. We have already provided an algorithm to compute ϑ_i for $i\geq 2$ in the main text by using (33). An alternative way is to back out the threshold value ϑ_i from the corresponding transversality conditions for any $i\geq 2$. It can be verified that all these values are the same for both algorithms, and that all these threshold values are independent of K_0. In other words, K_0 only has level effect (i.e. it only affects $C(0)$) but no speed effect on industrial upgrading. The main reason is that this economy is perfectly stationary, thus different initial capital levels only translate into different initial aggregate consumption and initial industrial structures.

7.4 Appendix 4

This appendix is to illustrate that the assumption of Leontief production function is not crucial for the main qualitative results. Suppose the production function is Cobb-Douglass for any industry level $i=1,2,...$ That is,

$$Y_i=A_iK_i^{\alpha_i}L_i^{1-\alpha_i},$$

where capital share α_i strictly increases with i but always belongs to interval $[0,1]$. The total output is $\sum_i Y_i$. Consider the simplest static case in which there are only two industries: $i=1$ and 2. The total labor and capital endowment are L and E. We can easily prove the following proposition:

Define

$$k_1^* \equiv \left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_2}\left(\frac{A_1}{A_2}\right)\right]^{\frac{1}{\alpha_2-\alpha_1}},$$
$$k_2^* \equiv \left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_1}\left(\frac{A_1}{A_2}\right)\right]^{\frac{1}{\alpha_2-\alpha_1}}.$$

(a) If $\frac{E}{L}\leq k_1^*$, the economy only produces good 1 with the total output $F\left(E,L\right)=A_1E^{\alpha_1}L^{1-\alpha_1}$.

(b) If $k_1^* < \frac{E}{L} < k_2^*$, then both goods will be produced with

$$\begin{aligned} L_1^* &= \left(1 - \frac{\frac{E}{L} - k_1^*}{k_2^* - k_1^*}\right) L,\ L_2^* = \frac{\frac{E}{L} - k_1^*}{k_2^* - k_1^*} L; \\ K_1^* &= k_1^* L_1^*,\ K_2^* = k_2^* L_2^*. \end{aligned}$$

The implied aggregate production function is

$$F(E, L) = \left[\begin{array}{c} \left(A_1 (k_1^*)^{\alpha_1} - \frac{A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}}{k_2^* - k_1^*} k_1^* \right) L \\ + \frac{A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}}{k_2^* - k_1^*} E \end{array} \right].$$

(c) If $\frac{K}{L} \geq k_2^*$, the economy only produces good 2 with total output $F(E, L) = A_2 E^{\alpha_2} L^{1-\alpha_2}$.

In addition, the aggregate production function $F(E, L)$ is constant return to scale, continuously differentiable, strictly monotone and concave. This result can be most clearly seen in the following figure, which plots the output per worker ($y = \frac{F(E,L)}{L}$) against the capital per worker ($k = \frac{E}{L}$).

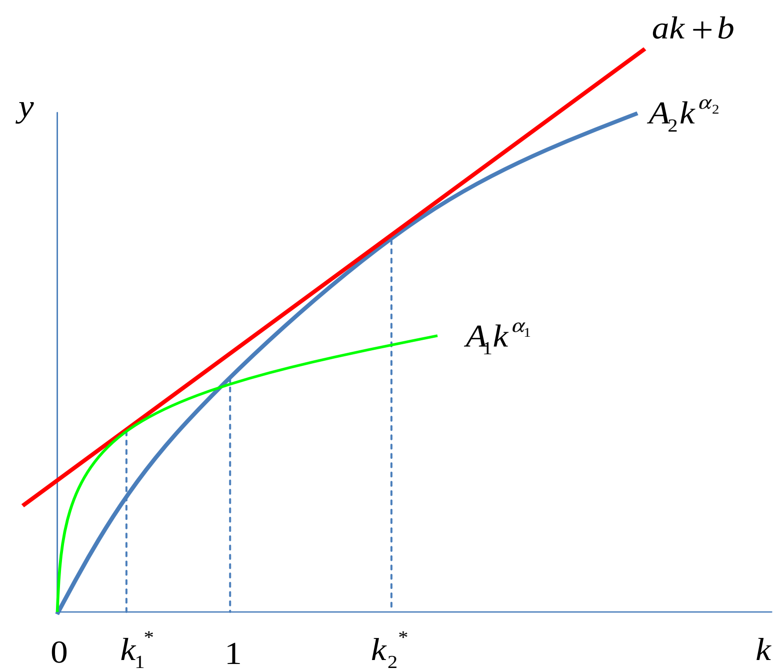

Figure 7: Structural Change with Cobb-Douglass Production Function

The straight line $y = ak + b$ is tangent to the production function curve $y = A_1 k^{\alpha_1}$ when $k = k_1^*$, and tangent to curve $y = A_2 k^{\alpha_2}$ when $k = k_2^*$. So when $k \in (k_1^*, k_2^*)$, the aggregate production function is just given by the cotangent line. Moreover, the slope a is nothing but the interest rate, which equals $\frac{A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}}{k_2^* - k_1^*}$, and the intercept b is nothing but the wage rate, which equals $A_1 (k_1^*)^{\alpha_1} - \frac{A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}}{k_2^* - k_1^*} k_1^*$.

So the interest rate and wage rate both keep constant when both good 1 and good 2 are produced. The same logic can be extended to the case with an infinite-dimensional commodity space, in which case the aggregate production function is simply the convex envelope of all the individual industry production functions although those infinitely many tangent points seem harder to analytically characterize in a general and very neat way as in the Leonteif production function case. This problem is also carried into the dynamic analysis, but it should be clear that the main qualitative results would still remain valid.

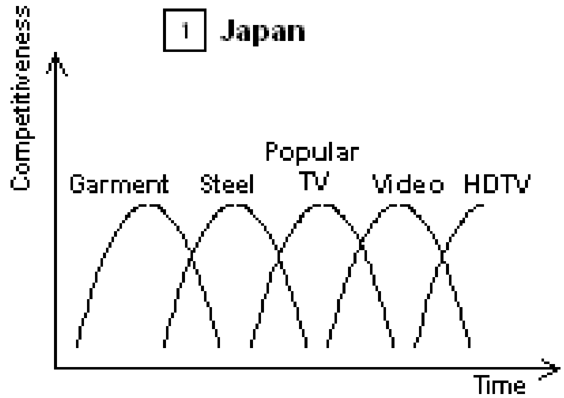

Figure 1: Japan's Industrial Upgrading and Product Evolution (quoted from Kojima, 2000)

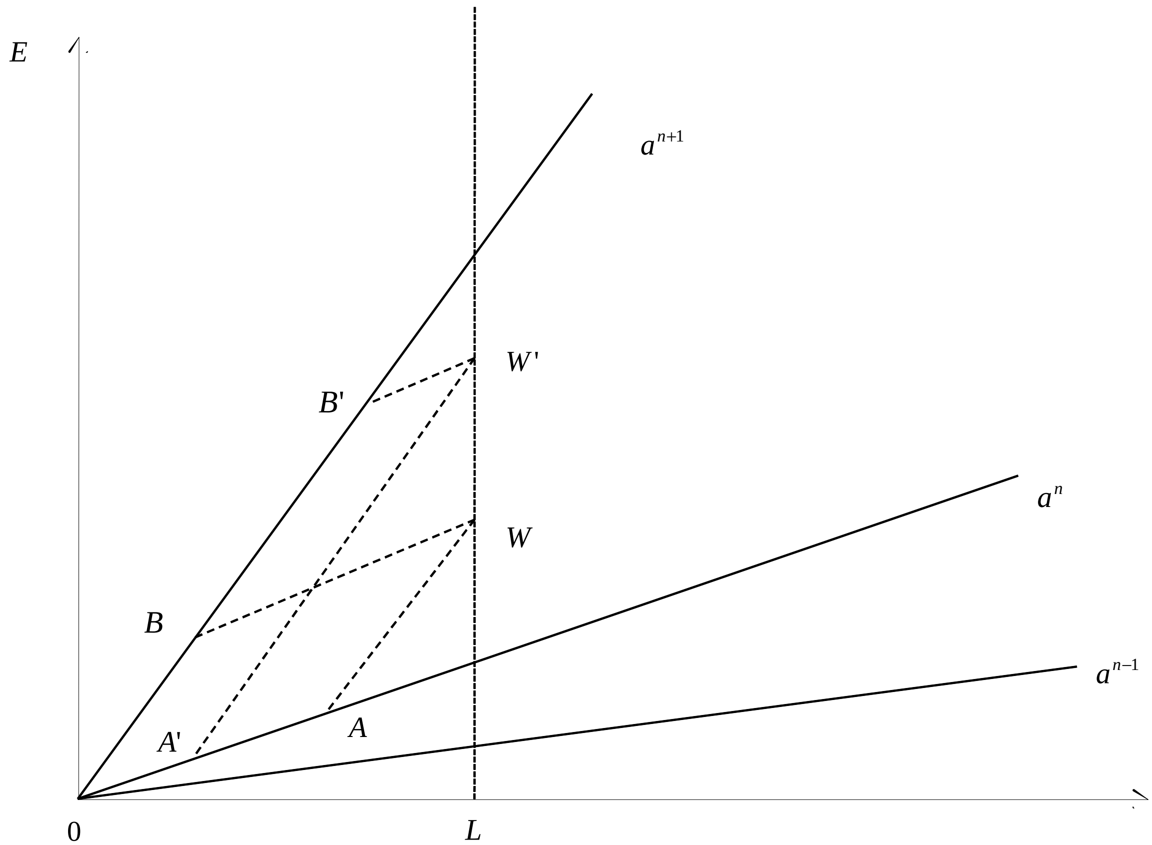

Figure 2. How Endowment Stucture Determines Optimal Industries

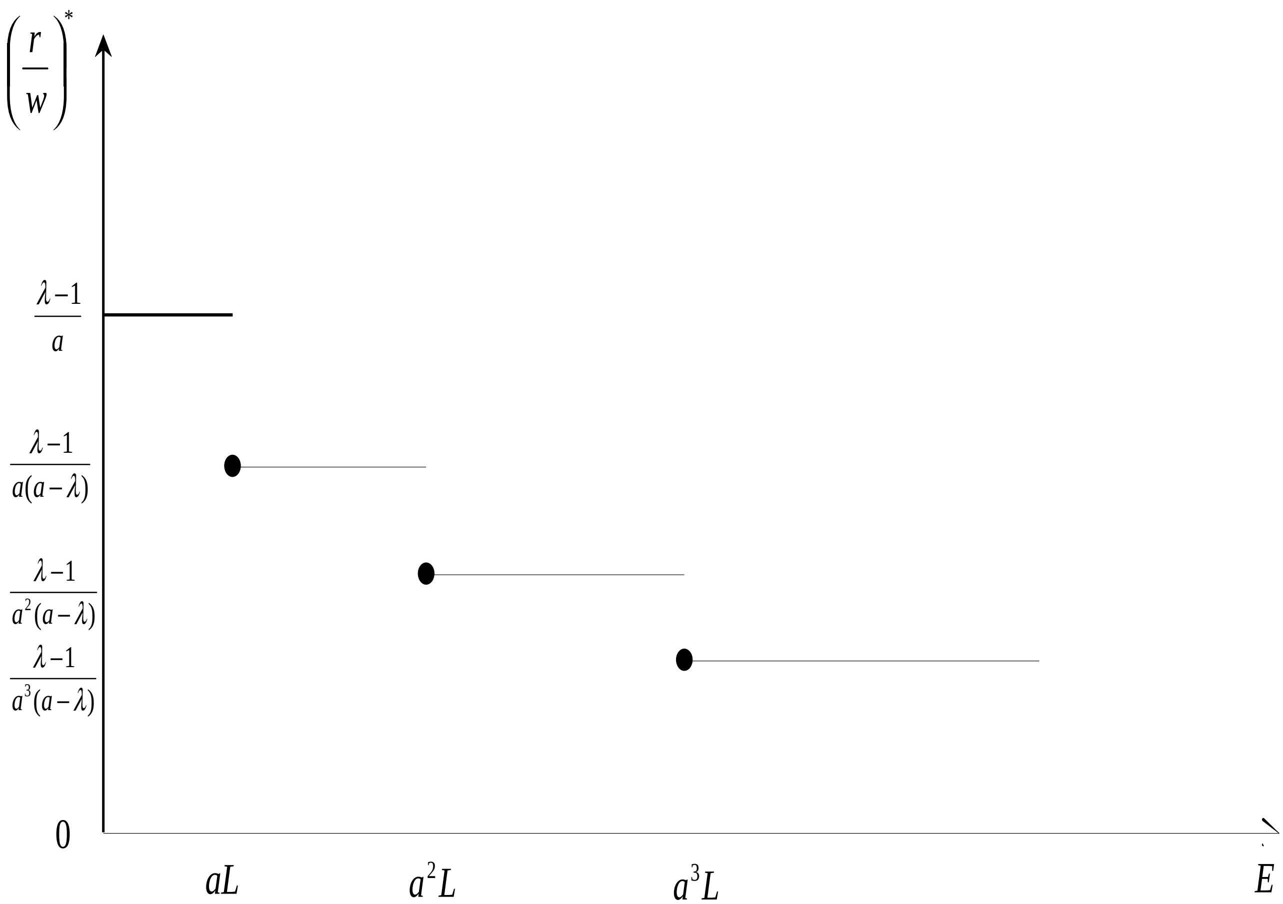

Figure 3. How Relative Factor Price Ratios Depend on Endowment Structure

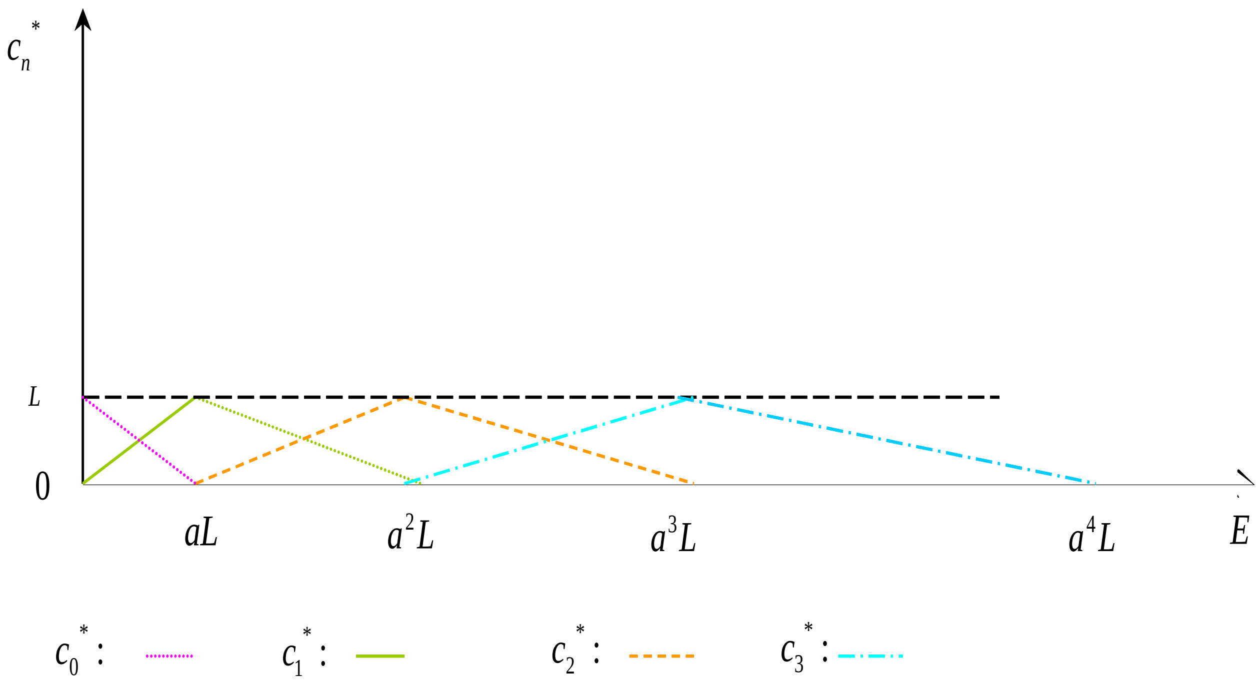

Figure 4. How Each Industry Depends on the Endowment

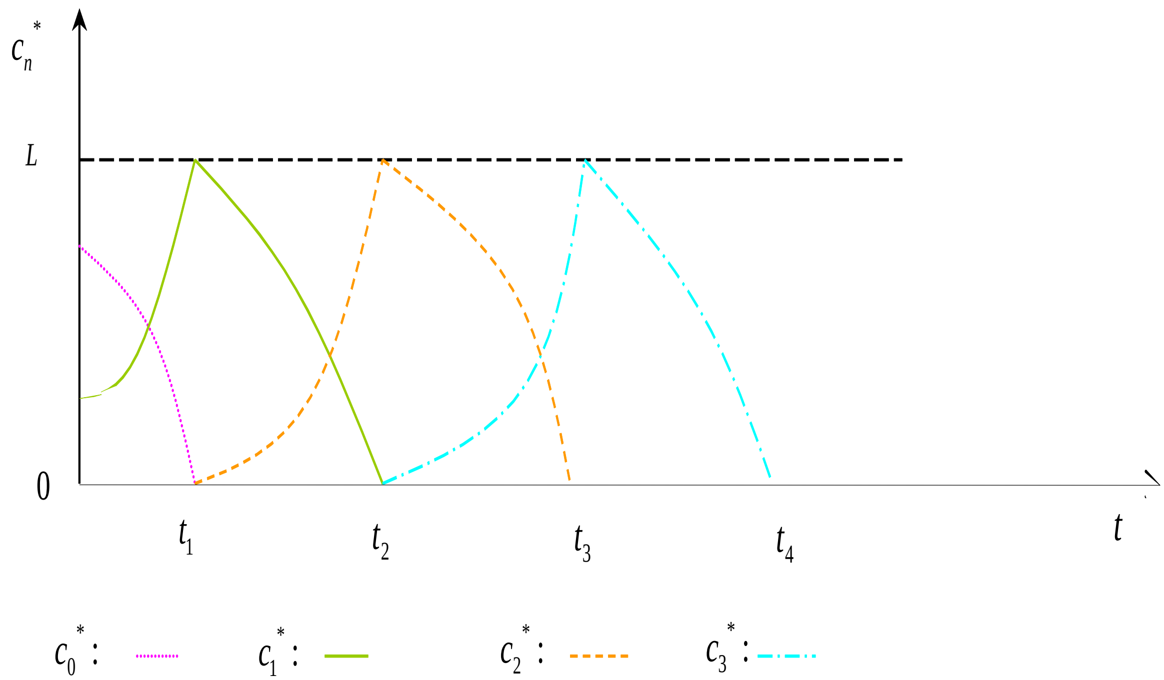

Figure 5. How Industries Evolve over Time when $K_0 \in (\vartheta_0, \vartheta_1)$

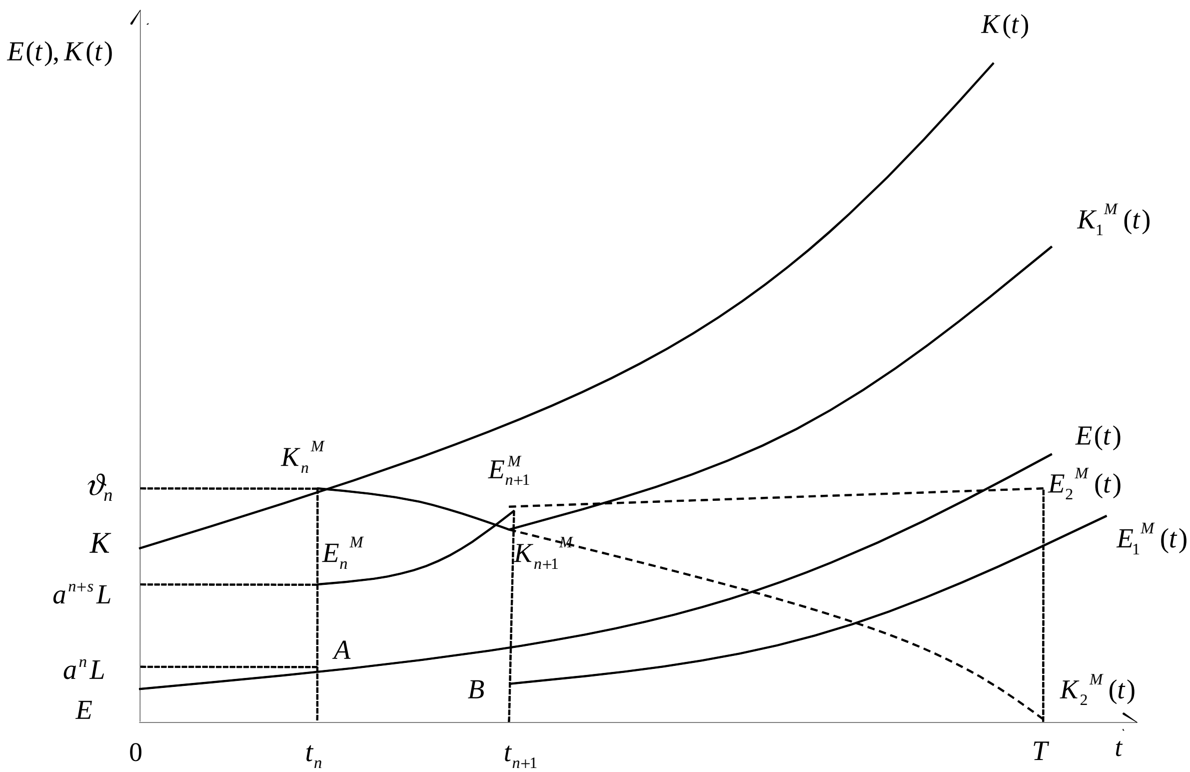

Figure 6. What Happens if Wrong Industrial Choice is Made