Introduction

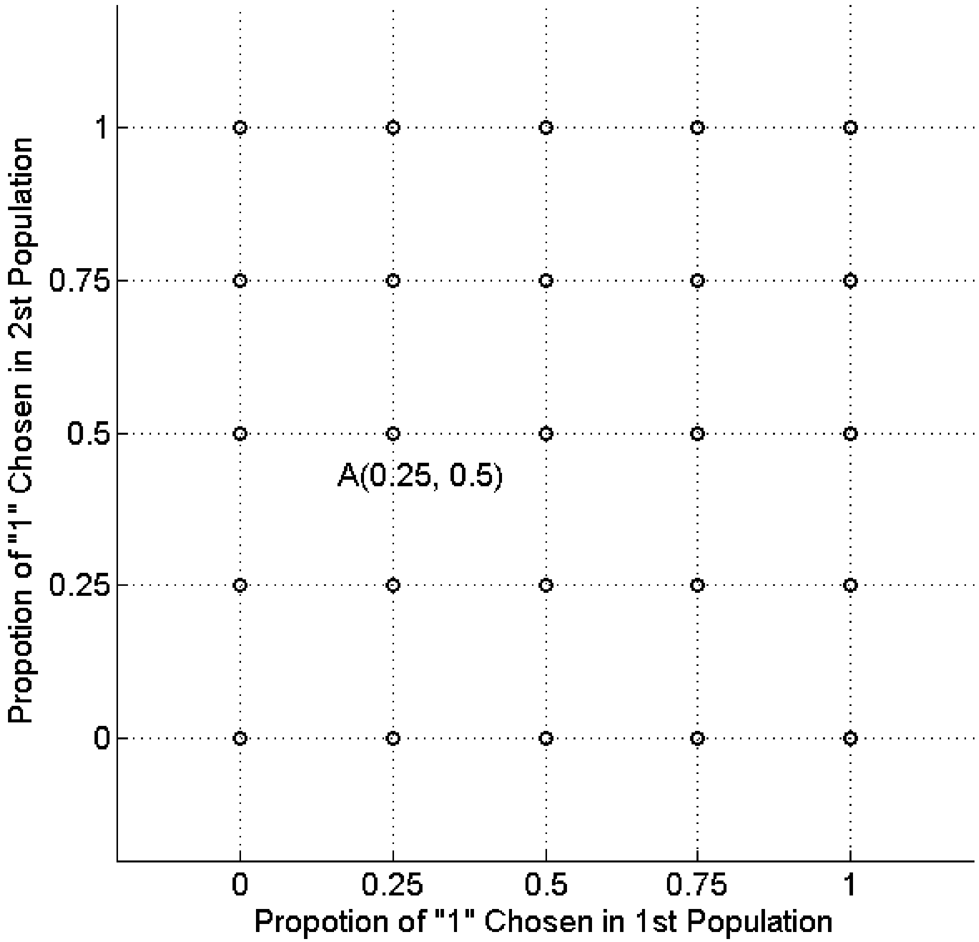
Experiment

-- It is a 2 populations Matching Pennies Game

	L	R
U	$5,0$	$0,5$
D	$0,5$	$5,0$

Only 1 treatment with 12 sessions.
8 subjects/Session (4 Up-Down and 4 Right-Left, 2 pop.)
Repeat 300 periods/Session
Anonymously in pairs in random matching
Last 1.5 – 2 hours each session
Mean income: 57.5 RMB/subject
In experimental Social science laboratory , ZJU
Conducted date: Oct.24-25, 2010

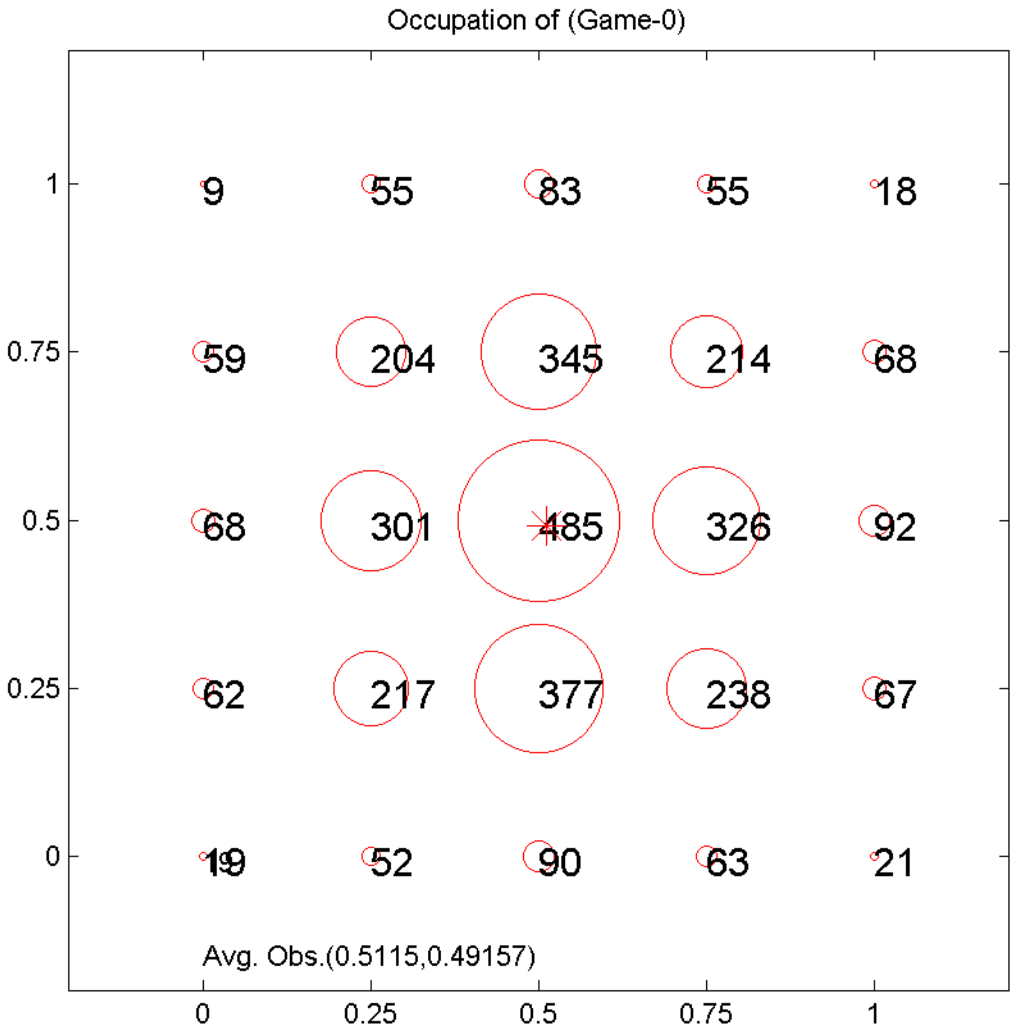
Observation of Distribution

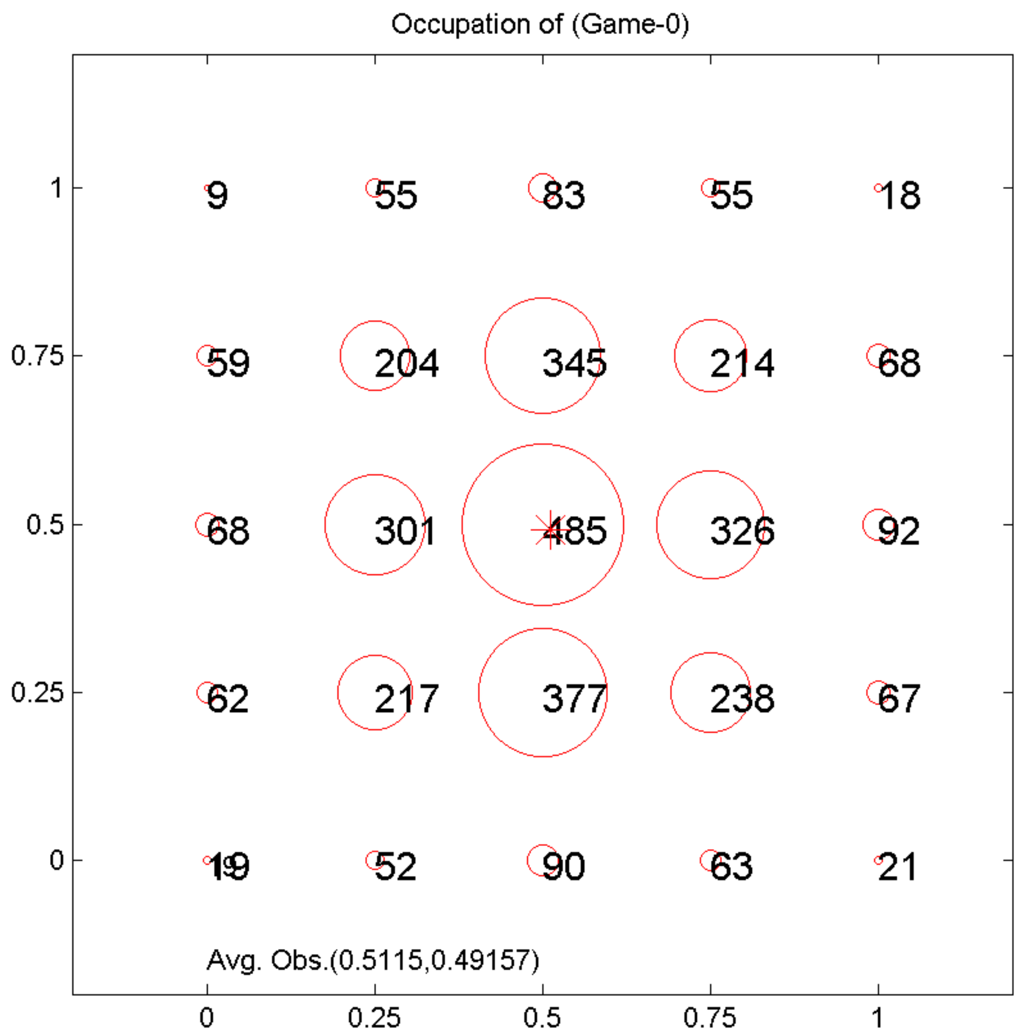


Example:
An observation at A (0.25,0.5) state means that for the 1*st* population, 0.25 of them choose 1 strategy and 0.75 choose 0 strategy; at the same time, for the 2*nd* population, two strategies are chosen half to half.

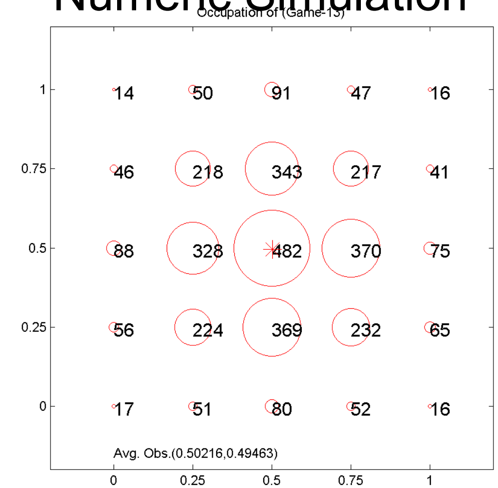
12 sessions
300 rounds/session
= **3600 obs**.
In the 5x5 lattices

Result(0)


Occupation of (Game-0)
9 55 83 55 18
59 204 345 214 68
68 301 485 326 92
62 217 377 238 67
19 52 90 63 21
Avg. Obs.(0.5115,0.49157)
0 0.25 0.5 0.75 1
1 0.75 0.5 0.25 0



Distribution of Numeric Simulation

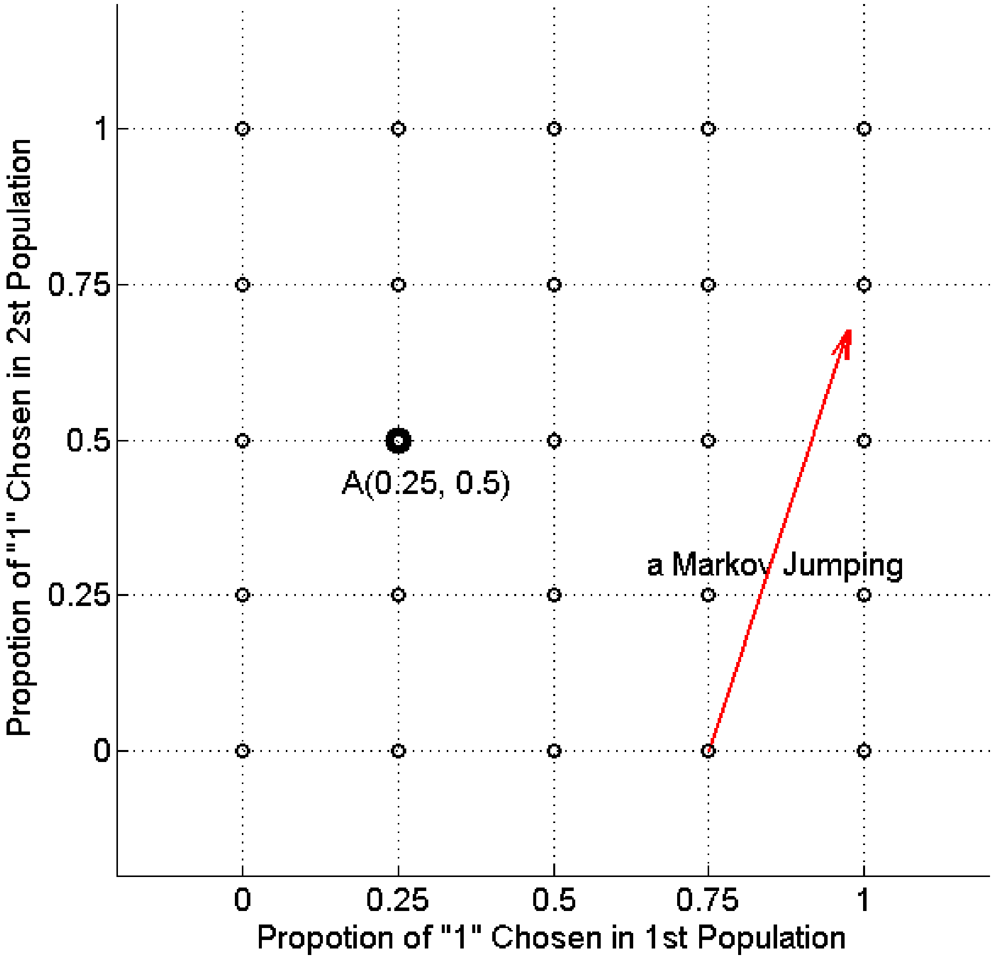


Theo. Prediction
(0.500, 0.500)
Normal distribution

14	56	84	56	14
56	225	338	225	56
84	338	506	338	84
56	225	338	225	56
14	56	84	56	14

What we have not seen yet?

Observation of Markov Jumping



Example:
An observation of Markov Jumping
Arrow here is a example for a Markov jumping, from state **(0.75, 0)** to its next round's state **(1, 0.75).**

12 sessions
300 rounds/session
have **3600 – 12**
= 3588 obs.
Of Markov Jumping
In the 5x5 lattices

Table 2: Markov Jumping Matrix of the 2 × 2 games with payoff matric [(5, 0)(0, 5); (0, 5)(5, 0)]

	$t+1$	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
t		0	0	0	0	0	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4
0	0	1	0	0	1	0	0	0	1	0	0	1	1	2	0	0	2	4	1	1	0	0	2	0	1	1
1	0	0	0	1	0	0	2	5	5	3	0	3	10	5	1	0	5	4	5	1	0	0	1	0	1	0
2	0	1	3	2	2	0	4	12	14	4	0	3	17	10	2	1	2	7	4	2	0	0	0	0	0	0
3	0	1	3	2	1	1	2	8	14	6	0	1	7	2	5	1	1	3	2	0	1	0	2	0	0	0
4	0	0	2	3	3	0	1	2	3	4	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0
0	1	0	0	1	0	0	2	2	3	0	0	4	8	9	2	0	8	12	10	0	0	0	0	0	0	1
1	1	1	3	1	1	2	4	15	23	11	1	8	27	31	8	1	8	22	22	7	1	3	10	5	2	0
2	1	5	8	7	6	1	14	37	40	23	2	10	44	56	27	7	9	18	36	11	2	1	3	5	5	0
3	1	5	6	13	6	3	7	25	39	11	2	9	19	28	21	3	1	4	20	9	2	0	2	2	0	1
4	1	2	3	10	2	0	0	5	10	8	4	0	5	5	5	3	0	1	1	2	1	0	0	0	0	0
0	2	0	2	0	0	0	2	3	2	0	0	1	8	7	2	1	4	9	8	6	0	1	3	6	2	1
1	2	0	2	2	1	3	5	16	26	8	3	6	32	37	20	8	6	33	47	16	5	2	7	11	5	0
2	2	0	8	18	5	3	6	28	48	24	5	8	41	76	50	13	3	30	55	35	6	0	5	10	6	2
3	2	3	6	11	8	4	8	26	41	37	13	4	18	46	36	4	3	9	22	14	5	0	4	3	1	0
4	2	0	3	4	6	1	0	6	13	17	3	0	6	12	7	5	0	2	4	2	0	0	0	0	1	0
0	3	0	0	0	0	0	1	1	3	2	0	1	3	4	5	0	2	3	10	11	1	0	3	5	3	1
1	3	0	0	2	2	0	1	3	15	7	2	4	16	28	21	5	1	15	30	23	4	1	2	14	8	0
2	3	0	0	8	5	0	1	12	27	17	7	3	23	65	45	13	2	14	34	32	14	0	4	9	8	2
3	3	0	2	4	6	1	1	5	27	30	12	1	9	24	35	11	1	5	12	9	12	0	0	4	2	1
4	3	0	0	0	5	1	1	2	10	7	3	0	0	9	12	4	0	1	4	7	0	0	0	1	1	0
0	4	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	0	0	1	1	2	0
1	4	0	0	0	2	0	0	0	1	3	2	1	0	5	9	0	0	2	6	5	5	0	3	4	3	4
2	4	0	0	1	0	0	0	4	4	6	4	0	3	17	4	5	1	4	8	14	2	1	2	2	1	0
3	4	0	1	0	0	1	0	1	6	7	2	0	2	6	10	5	0	3	3	3	4	0	0	0	1	0
4	4	0	0	0	1	0	0	0	3	2	2	0	1	2	0	1	0	0	2	2	1	0	0	0	1	0

(a)Full Markov Jumping(1:80)

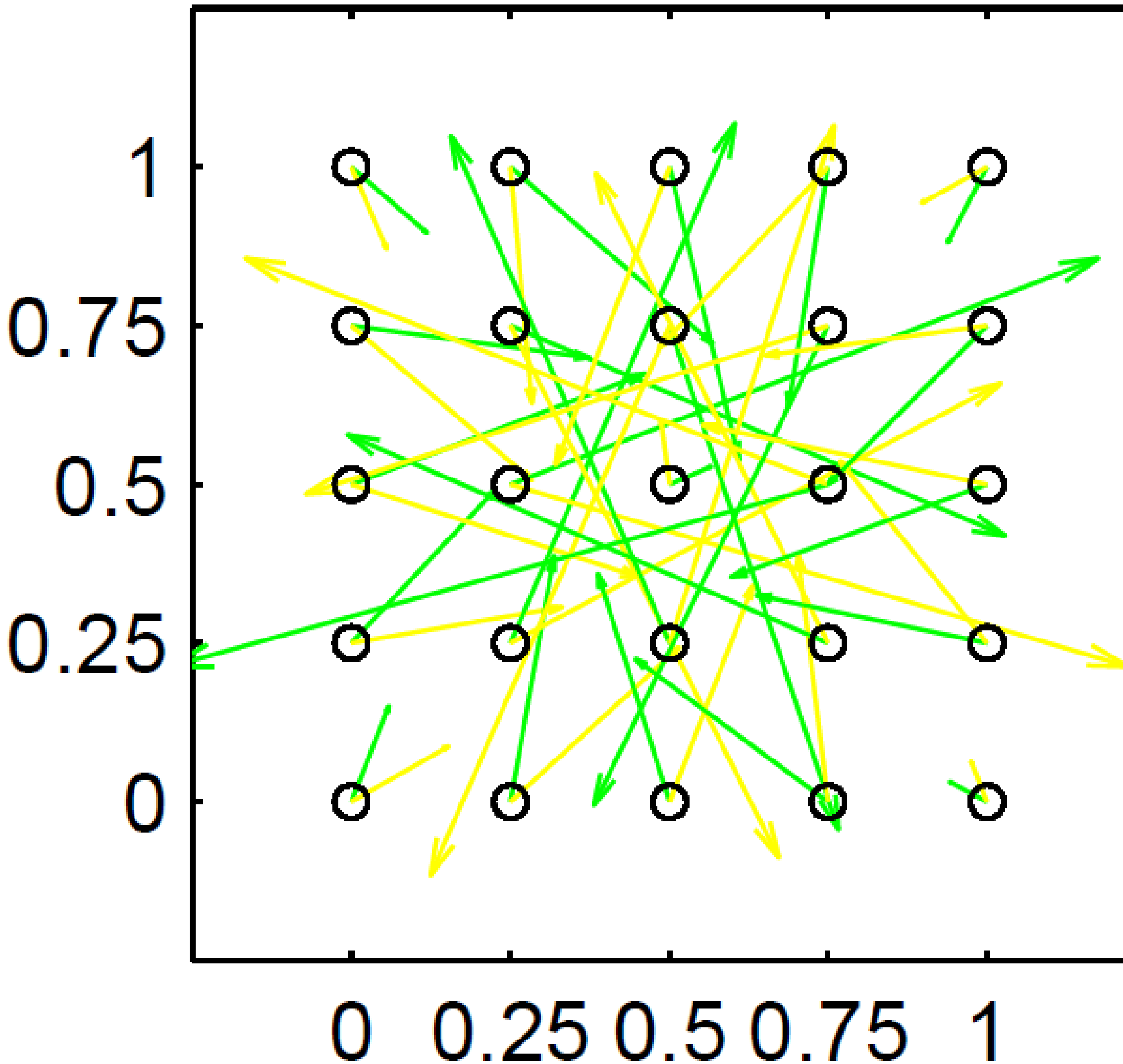


Table 2: Markov Jumping Matrix of the 2 × 2 games with payoff matric [(5, 0)(0, 5); (0, 5)(5, 0)]

	$t+1$	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	
t		0	0	0	0	0	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	
0	0	1	0	0	1	0	0	0	1	0	0	1	1	2	0	0	2	4	1	1	0	0	2	0	1	[illegible]
1	0	0	0	1	0	0	2	5	5	3	0	3	10	5	1	0	5	4	5	1	0	0	1	0	1	0
2	0	1	3	2	2	0	4	12	14	4	0	3	17	10	2	1	2	7	4	2	0	0	0	0	0	0
3	0	1	3	2	1	1	2	8	14	6	0	1	7	2	5	1	1	3	2	0	1	0	2	0	0	0
4	0	0	2	3	3	0	1	2	3	4	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0
0	1	0	0	1	0	0	2	2	3	0	0	4	8	9	2	0	8	12	10	0	0	0	0	0	0	1
1	1	1	3	1	1	2	4	15	23	11	1	8	27	31	8	1	8	22	22	7	1	3	10	5	2	0
2	1	5	8	7	6	1	14	37	40	23	2	10	44	56	27	7	9	18	36	11	2	1	3	5	5	0
3	1	5	6	13	6	3	7	25	39	11	2	9	19	28	21	3	1	4	20	9	2	0	2	2	0	1
4	1	2	3	10	2	0	0	5	10	8	4	0	5	5	5	3	0	1	1	2	1	0	0	0	0	0
0	2	0	2	0	0	0	2	3	2	0	0	1	8	7	2	1	4	9	8	6	0	1	3	6	2	1
1	2	0	2	2	1	3	5	16	26	8	3	6	32	37	20	8	6	33	47	16	5	2	7	11	5	0
2	2	0	8	18	5	3	6	28	48	24	5	8	41	76	50	13	3	30	55	35	6	0	5	10	6	2
3	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	0
4	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	0
0	3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	1
1	3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	16	28	21	5	1	15	30	23	4	1	2	14	8	0
2	3	0	0	8	5	0	1	12	27	17	7	3	23	65	45	13	2	14	34	32	14	0	4	9	8	2
3	3	0	2	4	6	1	1	5	27	30	12	1	9	24	35	11	1	5	12	9	12	0	0	4	2	1
4	3	0	0	0	5	1	1	2	10	7	3	0	0	9	12	4	0	1	4	7	0	0	0	1	1	0
0	4	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	1	0	0	1	1	2	0
1	4	0	0	0	2	0	0	0	1	3	2	1	0	5	9	0	0	2	6	5	5	0	3	4	3	4
2	4	0	0	1	0	0	0	4	4	6	4	0	3	17	4	5	1	4	8	14	2	1	2	2	1	0
3	4	0	1	0	0	1	0	1	6	7	2	0	2	6	10	5	0	3	3	3	4	0	0	0	1	0
4	4	0	0	0	1	0	0	0	3	2	2	0	1	2	0	1	0	0	2	2	1	0	0	0	1	0

(i',j')

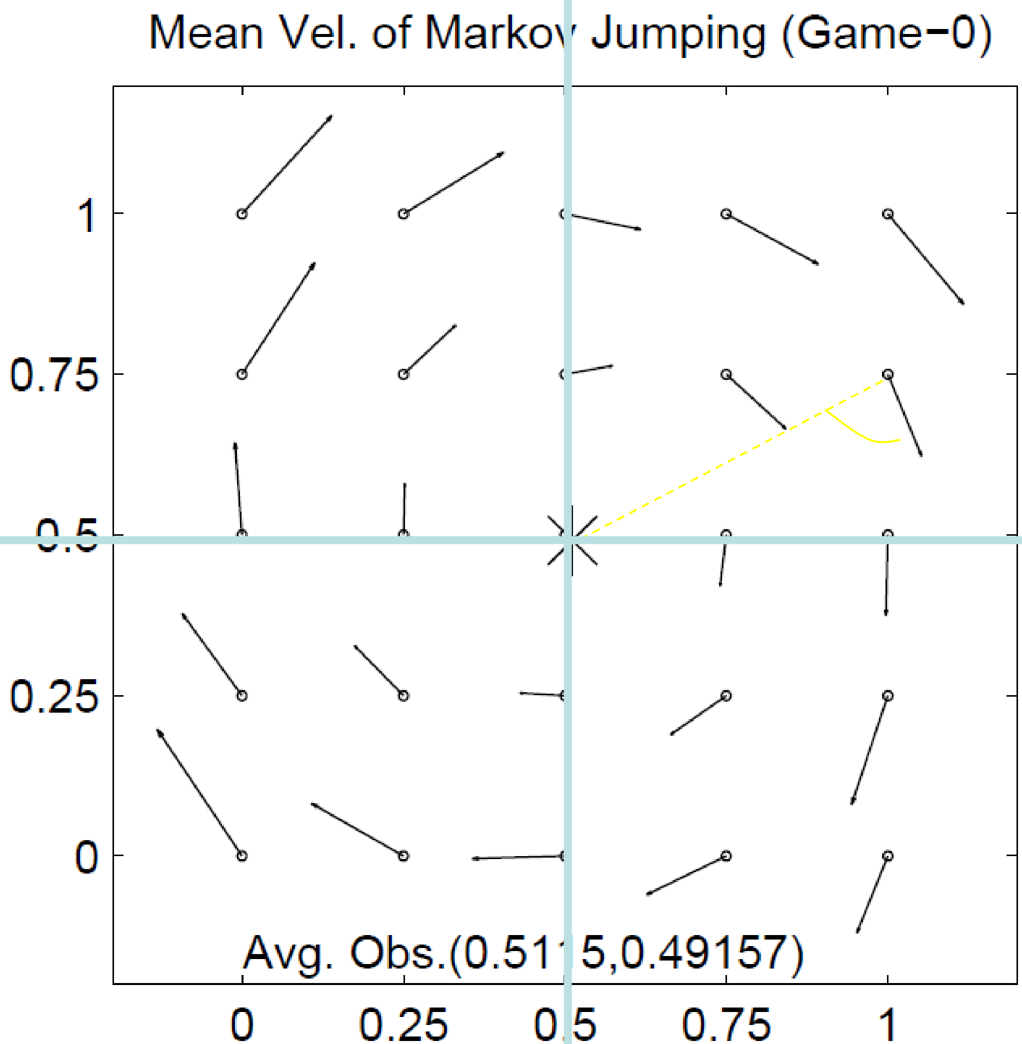
$A_{(i,j)\to(i',j')}\vec{J}_{(i,j)\to(i',j')}$

$A_{(i',j')\to(i,j)}\vec{J}_{(i',j')\to(i,j)}$

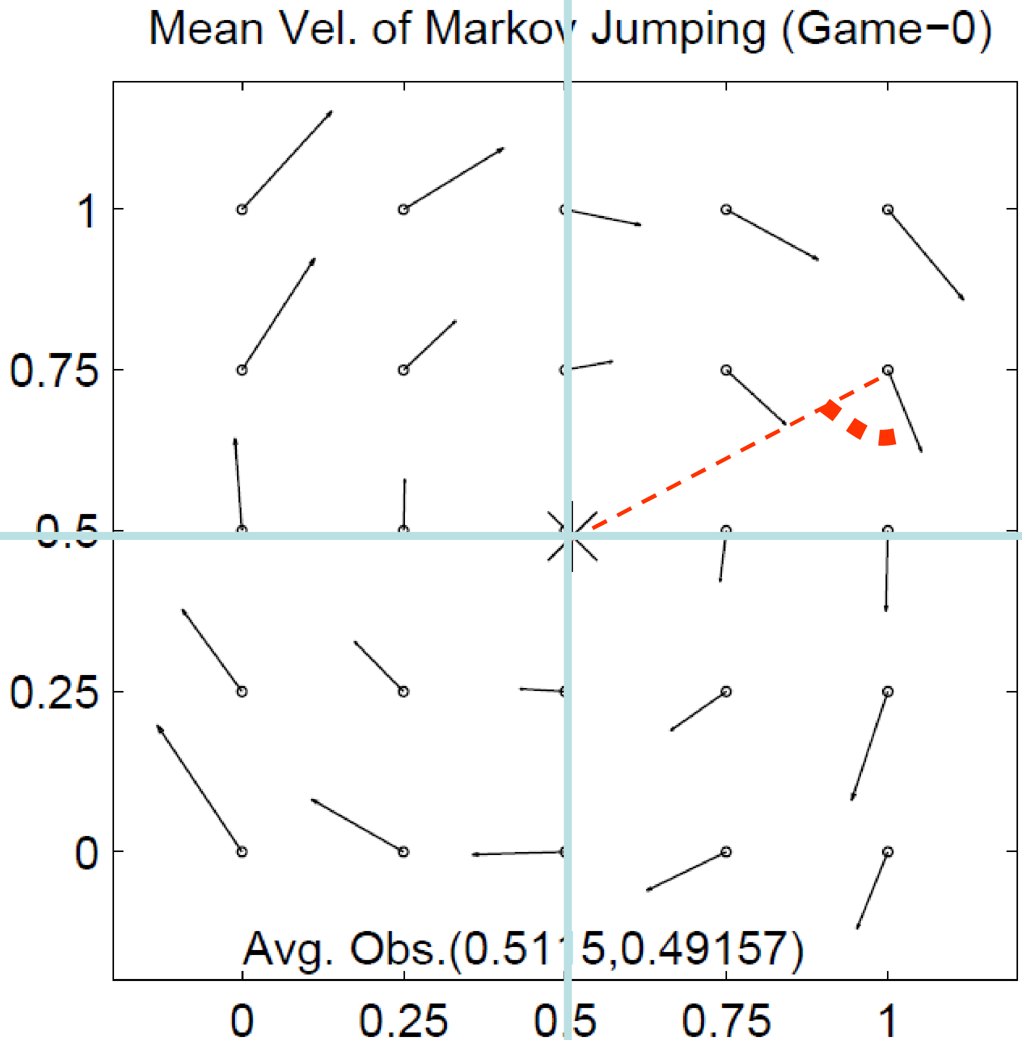
(i,j)

$$\vec{J}_{(i,j)\to(i',j')} = (i'-i)\vec{e}_1 + (j'-j)\vec{e}_2$$

$$\vec{v}_{(i,j)} = \frac{\sum_{(i',j')} \left(\boxed{A_{(i,j)\to(i',j')} \vec{J}_{(i,j)\to(i',j')}} + \boxed{A_{(i',j')\to(i,j)} \vec{J}_{(i',j')\to(i,j)}} \right)}{\sum_{(i',j')} A_{(i,j)\to(i',j')} + \sum_{(i',j')} A_{(i',j')\to(i,j)}} \qquad (1)$$



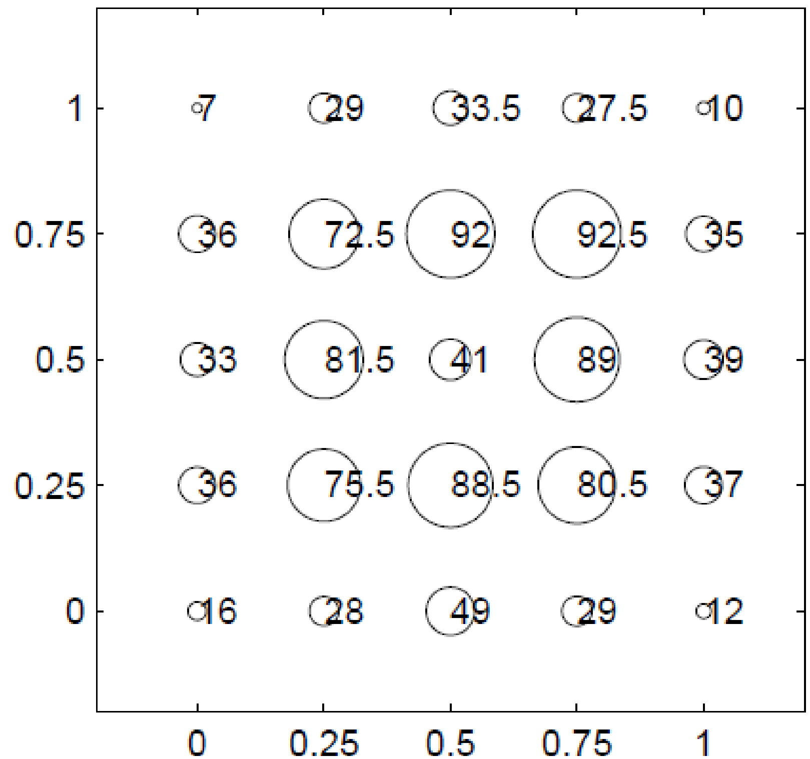
Linear Reg. Speed/Radius, cof.= 0.2714, *S.E.* = 0.0013 , *p* =0.000)
Mean Angular = 90.5 ± 1.9◦



90.5 ± 1.9◦

Linear Reg. Speed/Radius, cof.= 0.2714, *S.E.* = 0.0013 , *p* =0.000)
Mean Angular = 90.5 ± 1.9◦

Unbalanced Jumping Distribution



Ring-Mountain Pattern

$$f_{(i,j)} = \frac{1}{2} \sum_{(i',j')} |A_{(i',j')\to(i,j)} - A_{(i,j)\to(i',j')}|$$

Strictly Stationary Process Test

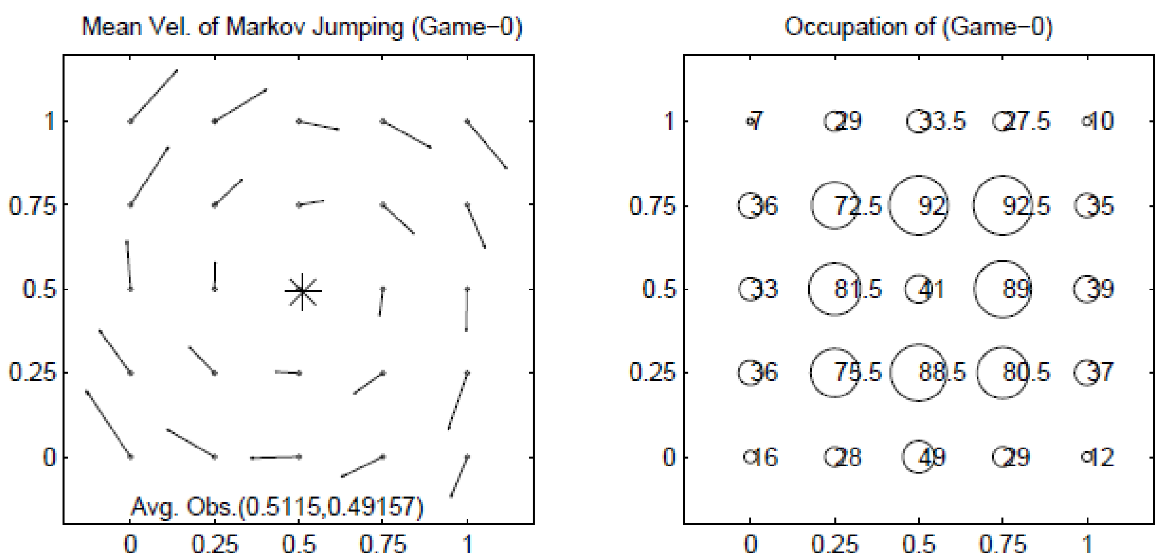
TABLE IV: Occupation unbalanced observation within strategy states in 12 sessions and $t - test(p < 0.01)$ result

$GameID$	ES	AS	$S1$	$S2$	$S3$	$S4$	$S5$	$S6$	$S7$	$S8$	$S9$	$S10$	$S11$	$S12$	$Reject0$	$p - value$	$Lbound$	$Ubound$	$method$
4	22	999	-2	-2	-6	-3	1	1	-6	-3	-6	1	-3	-4	1	0.004	-4.34	-0.98	$ss61320$
4	12	999	6	1	2	-8	4	6	7	5	4	7	8	8	1	0.007	1.353	6.980	$ss100200$
5	20	999	0	1	1	1	0	1	0	0	1	0	1	0	1	0.006	0.168	0.831	$ss100200$

Density of state not changing during any [*t0, t1*]

Result

- In our data, we show
- (1) the velocity field of Markov jumping
- (2) the distribution of unbalanced jumping form a ring-mountain shape



Support Data

1, Seleten & Chmura AER(2008)

Stationary Concepts for Experimental 2x2-Games

2, Cason, Friedman & Wagener(2005)

The dynamics of price dispersion, or Edgeworth variations

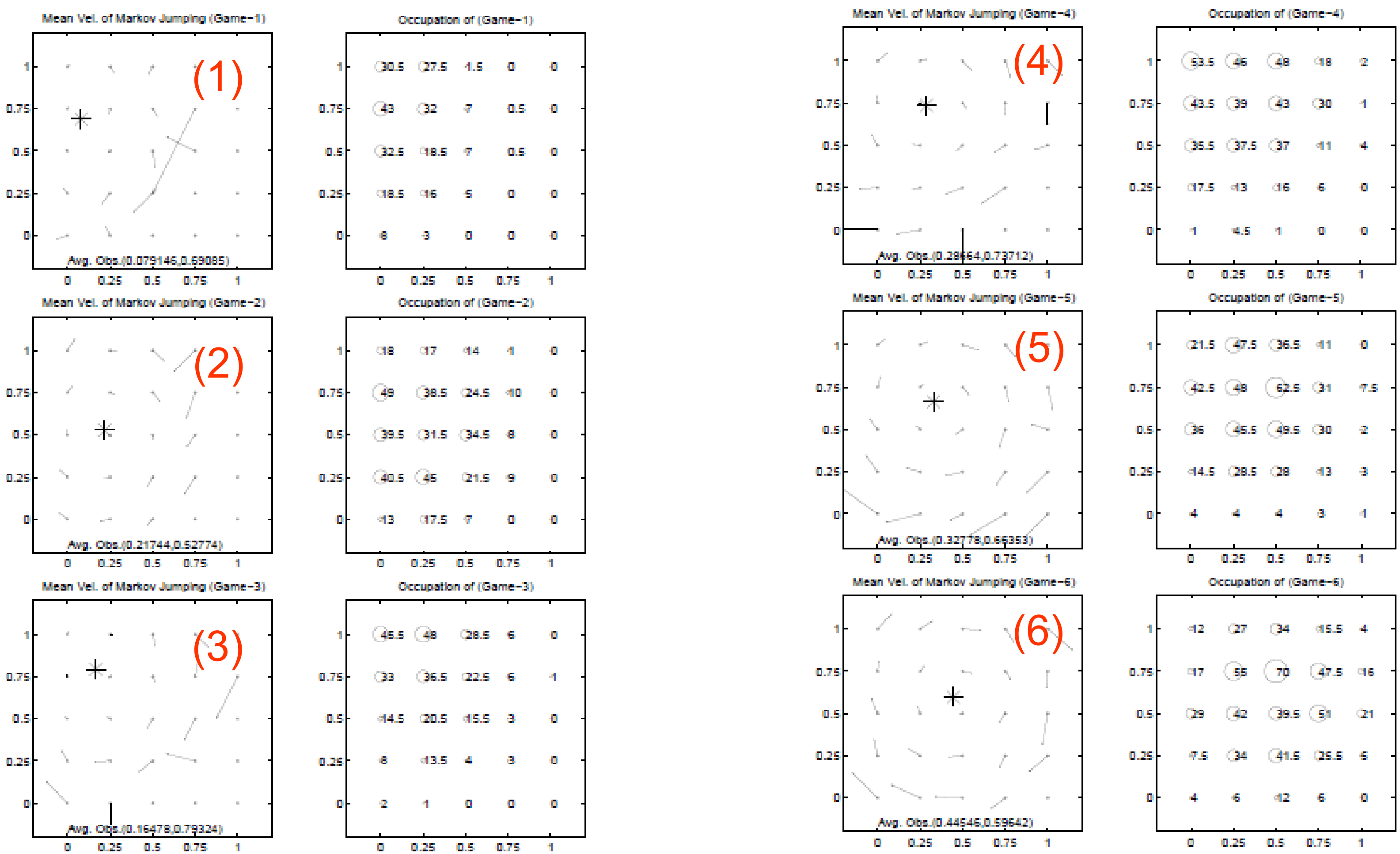
American Economic Review 2008, 98:3, 938–966
http://www.aeaweb.org/articles.php?doi=10.1257/aer.98.3.938

Stationary Concepts for Experimental 2x2-Games

By Reinhard Selten and Thorsten Chmura*

Table 3: Payoff matrix of Selten and Chmura 2×2 games

(1)	*L*	*R*	(7)	*L*	*R*
U	10, 8	0, 18	*U*	10, 12	4, 22
D	9, 9	10, 8	*D*	9, 9	14, 8
(2)	*L*	*R*	(8)	*L*	*R*
U	9, 4	0, 13	*U*	9, 7	3, 16
D	6, 7	8, 5	*D*	6, 7	11, 5
(3)	*L*	*R*	(9)	*L*	*R*
U	8, 6	0, 14	*U*	8, 9	3, 17
D	7, 7	10, 4	*D*	7, 7	13, 4
(4)	*L*	*R*	(10)	*L*	*R*
U	7, 4	0, 11	*U*	7, 6	2, 13
D	5, 6	9, 2	*D*	5, 6	11, 2
(5)	*L*	*R*	(11)	*L*	*R*
U	7, 2	0, 9	*U*	7, 4	2, 11
D	4, 5	8, 1	*D*	4, 5	10, 1
(6)	*L*	*R*	(12)	*L*	*R*
U	7, 1	1, 7	*U*	7, 3	3, 9
D	3, 5	8, 0	*D*	3, 5	10, 0



Mean Vel. of Markov Jumping (Game−1)
(1)
Avg. Obs.(0.079146,0.69085)
Occupation of (Game−1)
Mean Vel. of Markov Jumping (Game−2)
(2)
Avg. Obs.(0.21744,0.52774)
Occupation of (Game−2)
Mean Vel. of Markov Jumping (Game−3)
(3)
Avg. Obs.(0.16478,0.79324)
Occupation of (Game−3)
Mean Vel. of Markov Jumping (Game−4)
(4)
Avg. Obs.(0.28664,0.73712)
Occupation of (Game−4)
Mean Vel. of Markov Jumping (Game−5)
(5)
Avg. Obs.(0.32778,0.66353)
Occupation of (Game−5)
Mean Vel. of Markov Jumping (Game−6)
(6)
Avg. Obs.(0.44546,0.59642)
Occupation of (Game−6)

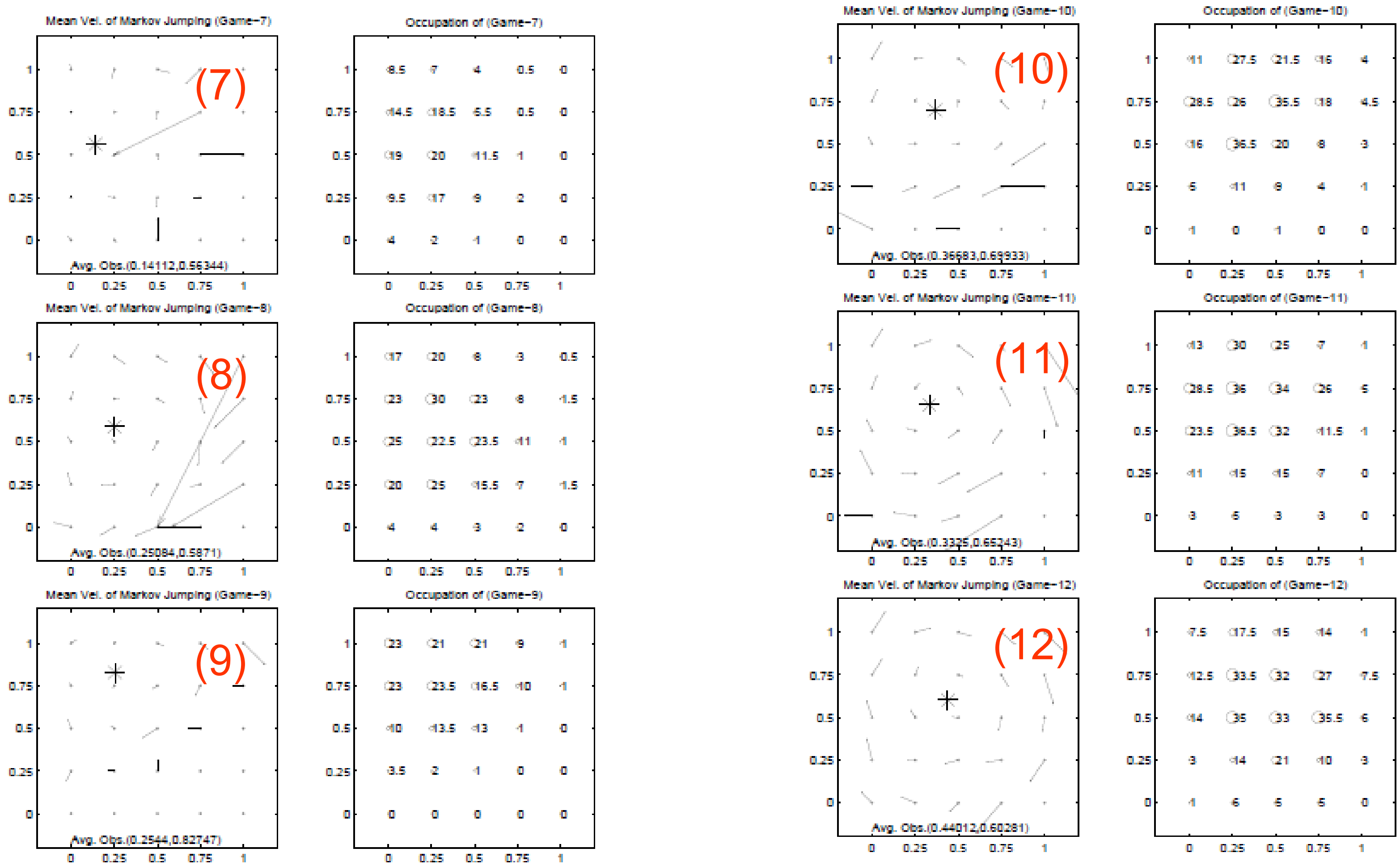

Mean Vel. of Markov Jumping (Game−7)
(7)
Avg. Obs.(0.14112,0.56344)
Occupation of (Game−7)
Mean Vel. of Markov Jumping (Game−8)
(8)
Avg. Obs.(0.25084,0.5871)
Occupation of (Game−8)
Mean Vel. of Markov Jumping (Game−9)
(9)
Avg. Obs.(0.2544,0.82747)
Occupation of (Game−9)
Mean Vel. of Markov Jumping (Game−10)
(10)
Avg. Obs.(0.36683,0.69933)
Occupation of (Game−10)
Mean Vel. of Markov Jumping (Game−11)
(11)
Avg. Obs.(0.3325,0.65243)
Occupation of (Game−11)
Mean Vel. of Markov Jumping (Game−12)
(12)
Avg. Obs.(0.44012,0.60281)
Occupation of (Game−12)

Table 1
Estimated transition matrices for *all* periods in experienced sessions

Observ.		Search cost = 20, $q=\frac{1}{3}$				Observ.		Search cost = 20, $q=\frac{2}{3}$			
		Q_1^{t+1}	Q_2^{t+1}	Q_3^{t+1}	Q_4^{t+1}			Q_1^{t+1}	Q_2^{t+1}	Q_3^{t+1}	Q_4^{t+1}
82	Q_1^t	0.73	0.11	0.09	0.07	83	Q_1^t	0.75	0.10	0.06	0.10
70	Q_2^t	0.24	0.59	0.13	0.04	63	Q_2^t	0.22	0.62	0.08	0.08
65	Q_3^t	0.08	0.25	0.58	0.09	75	Q_3^t	0.13	0.12	0.59	0.16
71	Q_4^t	0.03	0.11	0.14	0.72	61	Q_4^t	0.02	0.11	0.36	0.51
Observ.		Search cost = 60, $q=\frac{1}{3}$				Observ.		Search cost = 60, $q=\frac{2}{3}$			
		Q_1^{t+1}	Q_2^{t+1}	Q_3^{t+1}	Q_4^{t+1}			Q_1^{t+1}	Q_2^{t+1}	Q_3^{t+1}	Q_4^{t+1}
71	Q_1^t	0.63	0.18	0.07	0.11	74	Q_1^t	0.66	0.16	0.09	0.08
74	Q_2^t	0.24	0.55	0.14	0.07	71	Q_2^t	0.24	0.55	0.07	0.14
74	Q_3^t	0.04	0.24	0.62	0.09	71	Q_3^t	0.03	0.25	0.52	0.20
69	Q_4^t	0.14	0.06	0.16	0.64	72	Q_4^t	0.10	0.11	0.28	0.51

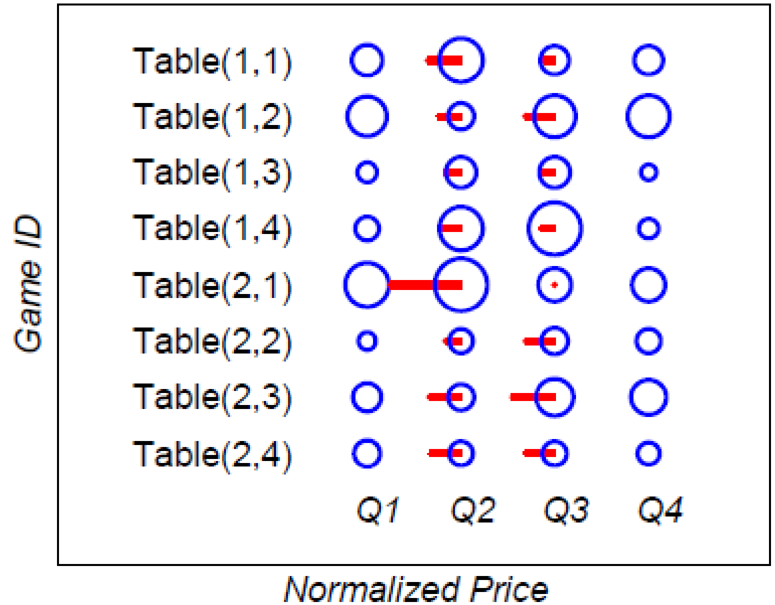


Figure 5: Velocity pattern in lattices for Game (1.1)-(2.4) respectively. game ID (a.b), a is the i.d. of Table, and b is the id of sub Table. Data from Cason's Transition Matrix in P815, Table 1 and Table 2 from Ref. Cason et al. (2005)

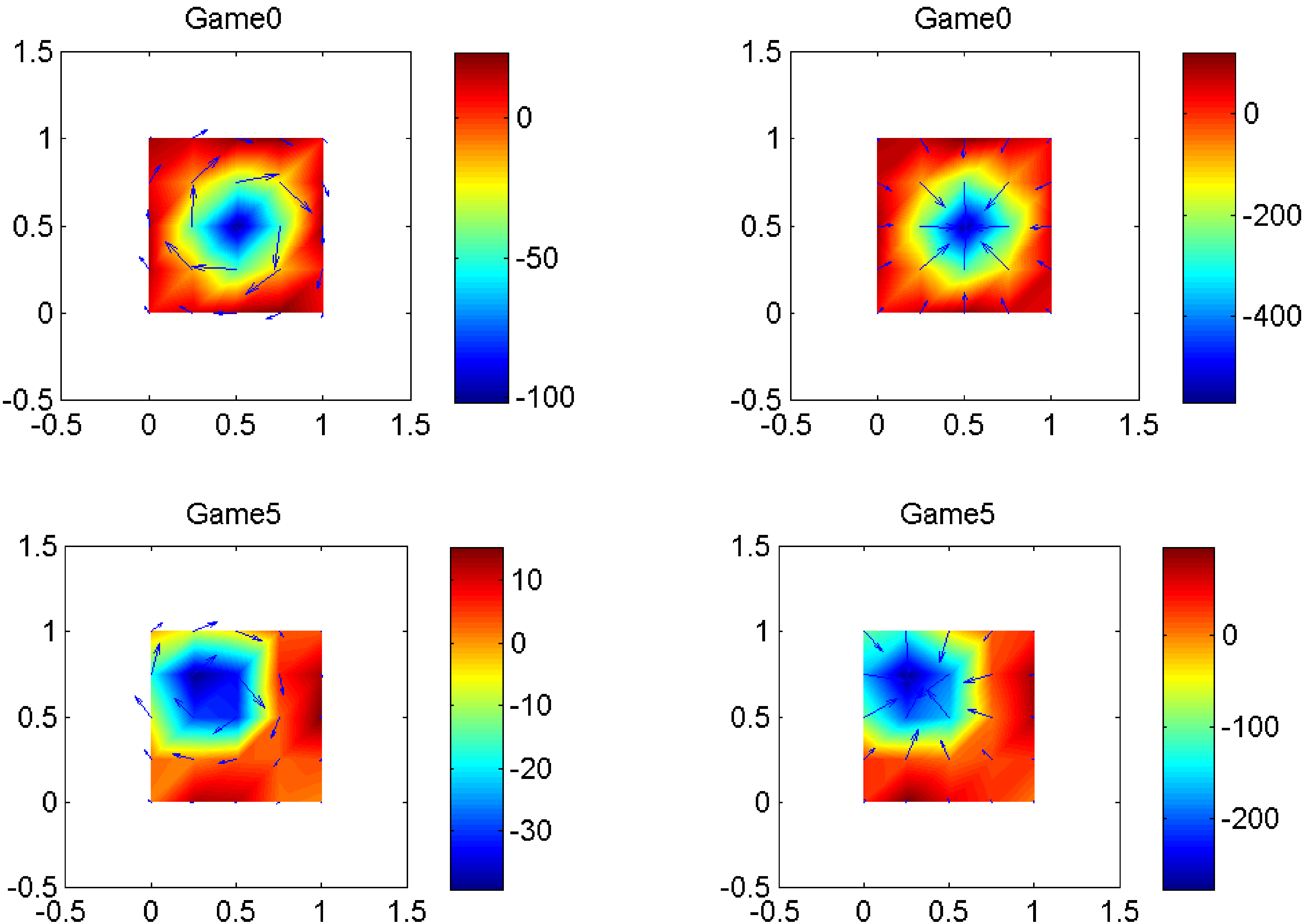
Conclusion

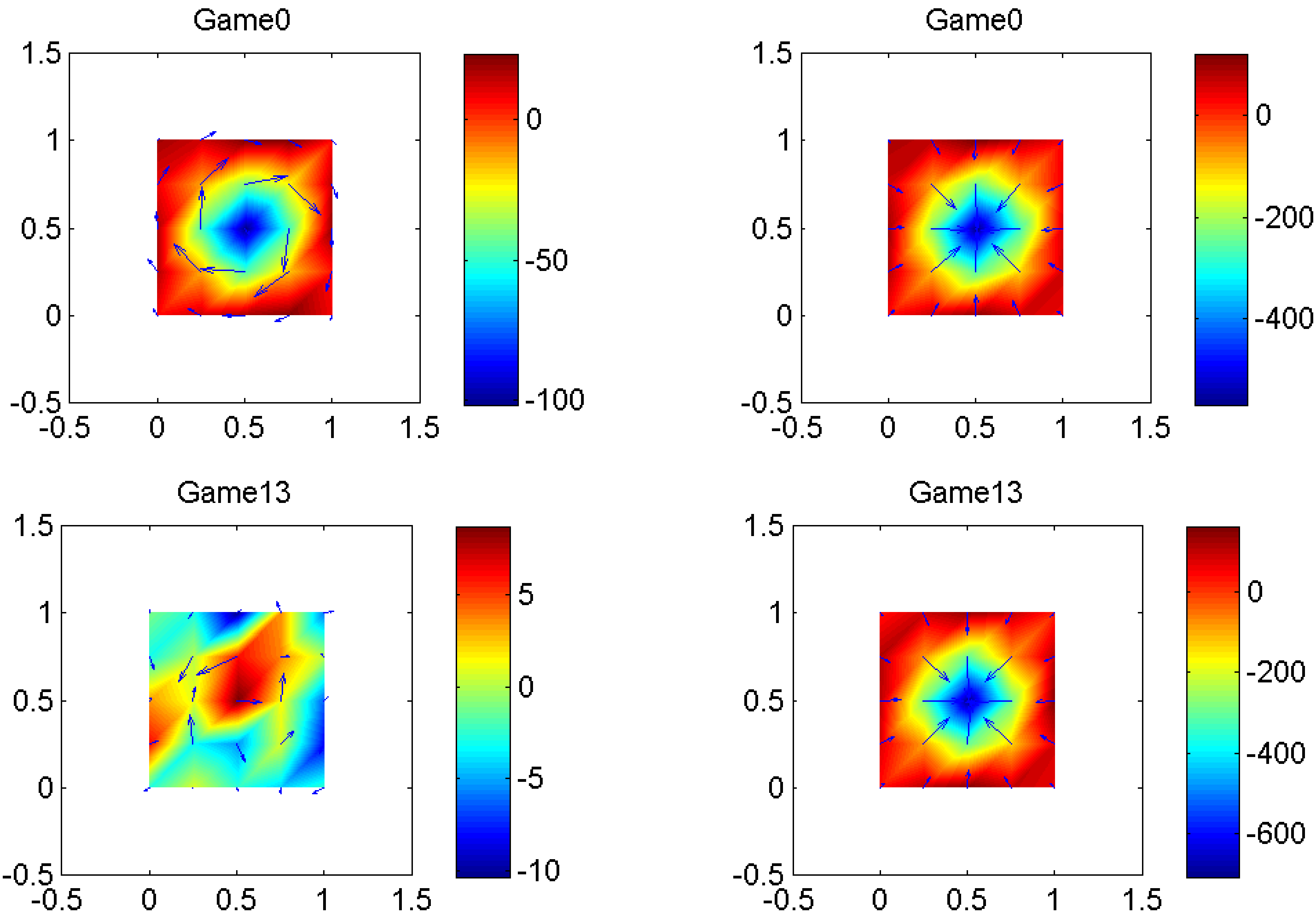
With laboratory experiments,
we obtain (firstly?)
a hidden order in systems
usually called as
“mixed strategy Nash equilibrium”

Bertrand-Edgeworth-Shapley Cycle in a 2 × 2 Game

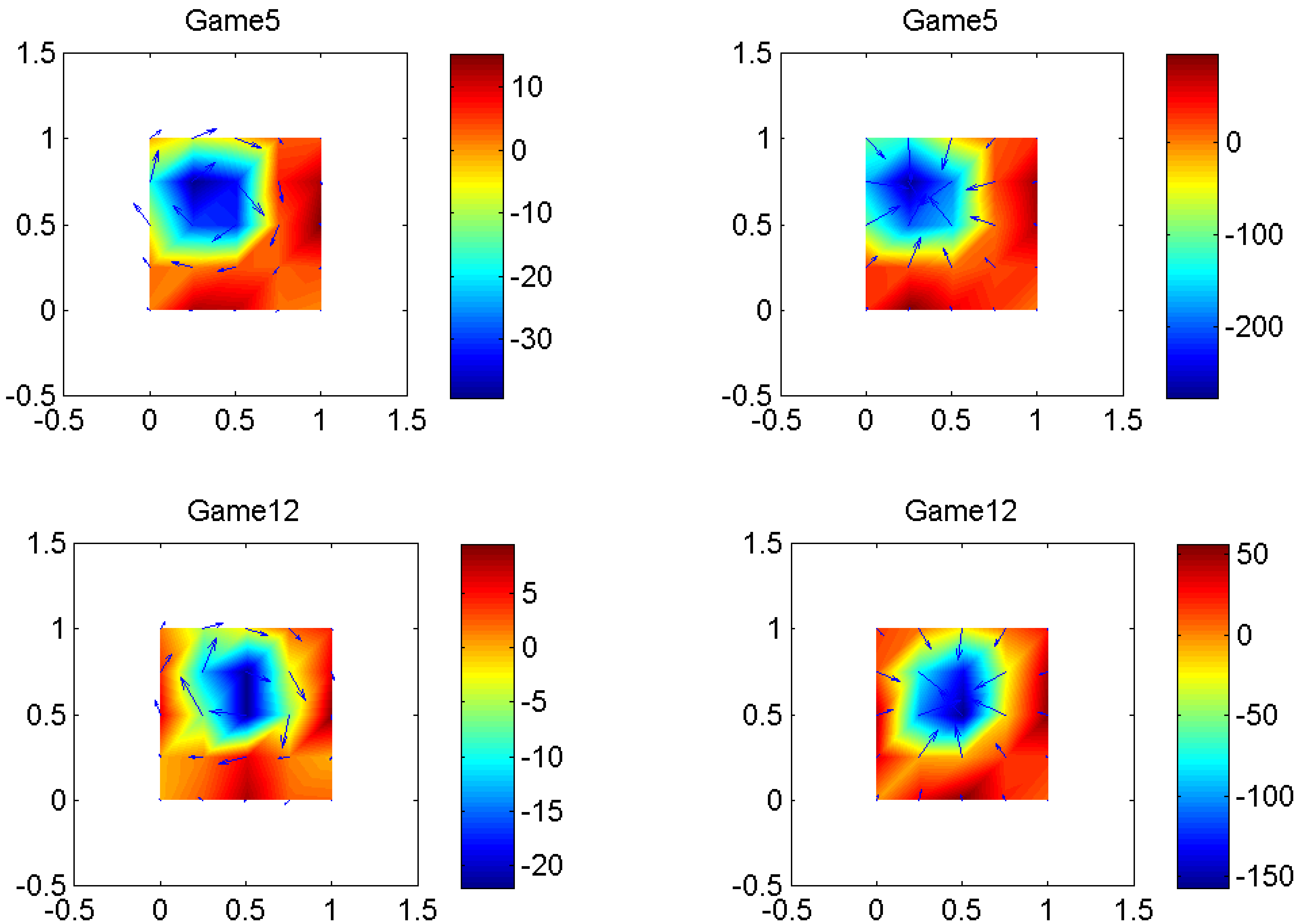
Bin Xu
Zhijian Wang

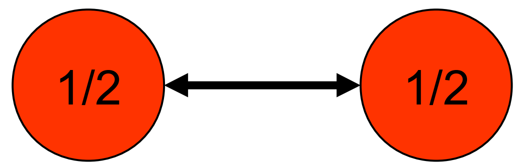
Dec 16, 2010 Xianmen, China

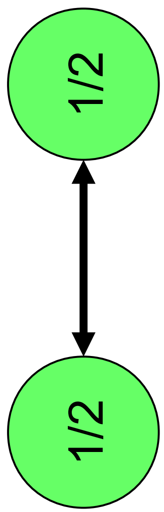

Game0
Game0
Game5
Game5
1.5
1
0.5
0
-0.5
-0.5
0
0.5
1
1.5
0
-50
-100
0
-200
-400
10
0
-10
-20
-30
0
-100
-200



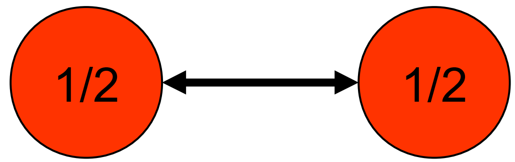
Game0
1.5
1
0.5
0
-0.5
-0.5
0
0.5
1
1.5
0
-50
-100
Game0
1.5
1
0.5
0
-0.5
-0.5
0
0.5
1
1.5
0
-200
-400
Game13
1.5
1
0.5
0
-0.5
-0.5
0
0.5
1
1.5
5
0
-5
-10
Game13
1.5
1
0.5
0
-0.5
-0.5
0
0.5
1
1.5
0
-200
-400
-600

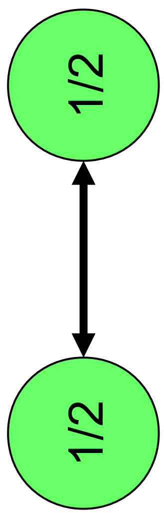

Game5
1.5
1
0.5
0
-0.5
-0.5
0
0.5
1
1.5
10
0
-10
-20
-30
Game5
1.5
1
0.5
0
-0.5
-0.5
0
0.5
1
1.5
0
-100
-200
Game12
1.5
1
0.5
0
-0.5
-0.5
0
0.5
1
1.5
5
0
-5
-10
-15
-20
Game12
1.5
1
0.5
0
-0.5
-0.5
0
0.5
1
1.5
50
0
-50
-100
-150



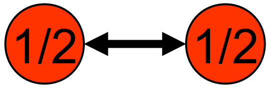


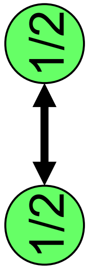
	L	R
U	$5,0$	$0,5$
D	$0,5$	$5,0$



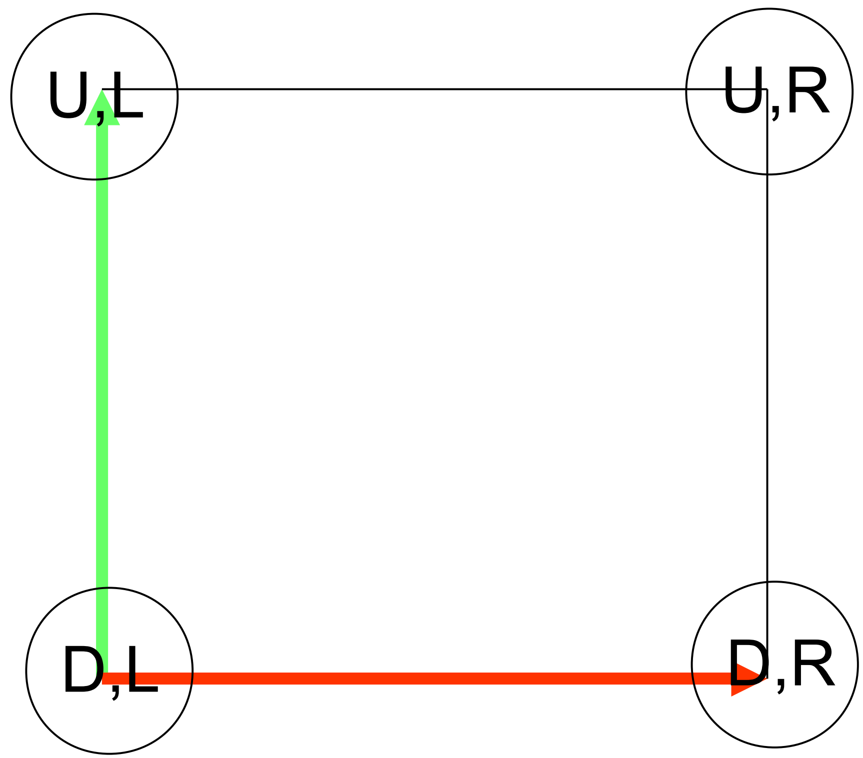


	L	R
U	$5,0$	$0,5$
D	$0,5$	$5,0$





	L	R
U	$5, 0$	$0, 5$
D	$0, 5$	$5, 0$



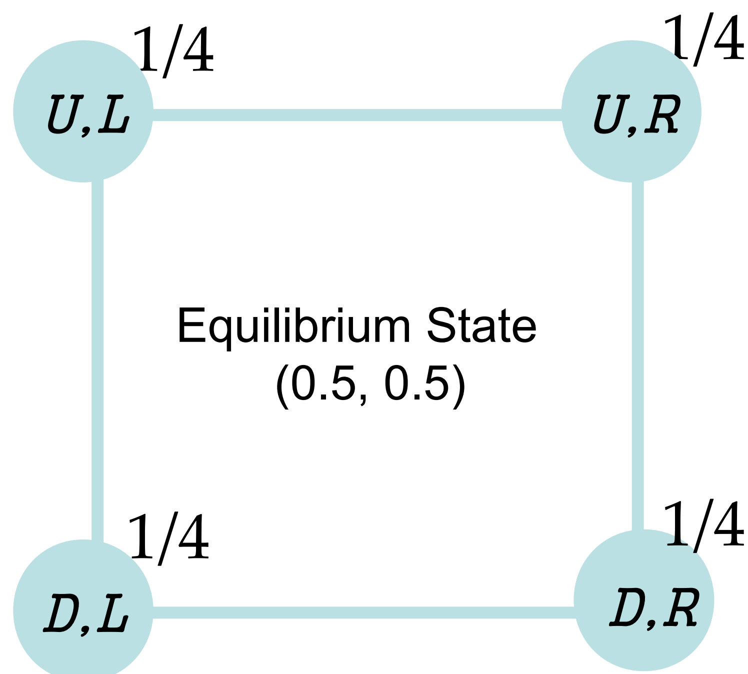
What we have known in the matching pennies game?

	L	R
U	$5,0$	$0,5$
D	$0,5$	$5,0$

What we have known in matching pennies game?

		1/2	1/2
		L	R
1/2	U	5, 0	0, 5
1/2	D	0, 5	5, 0

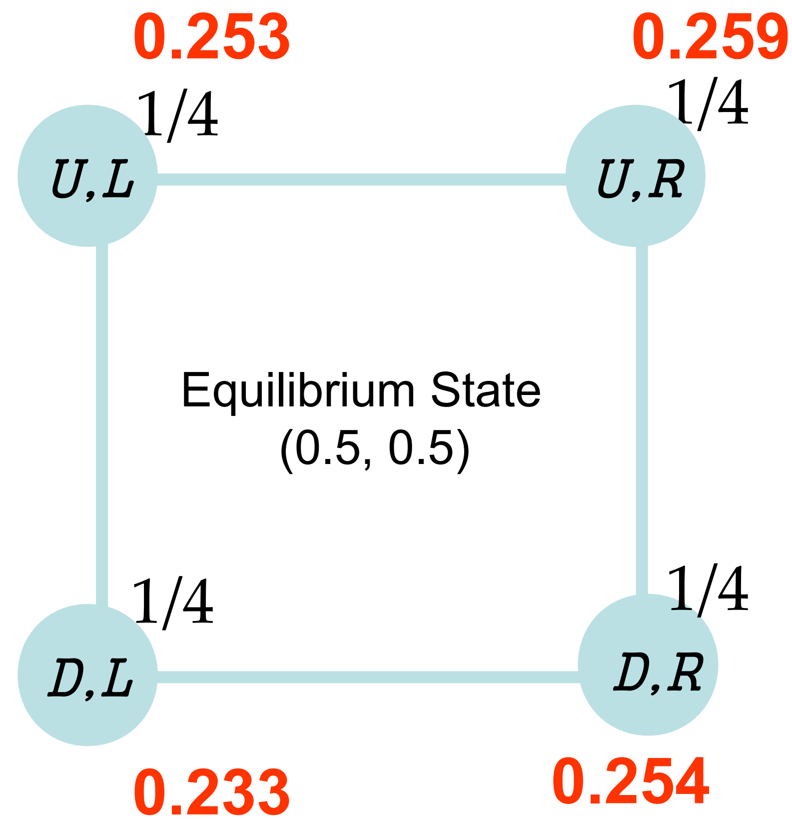
strategy possibility
0.5



What we have known in matching pennies game?

		1/2	1/2
		L	R
1/2	U	5, 0	0, 5
1/2	D	0, 5	5, 0

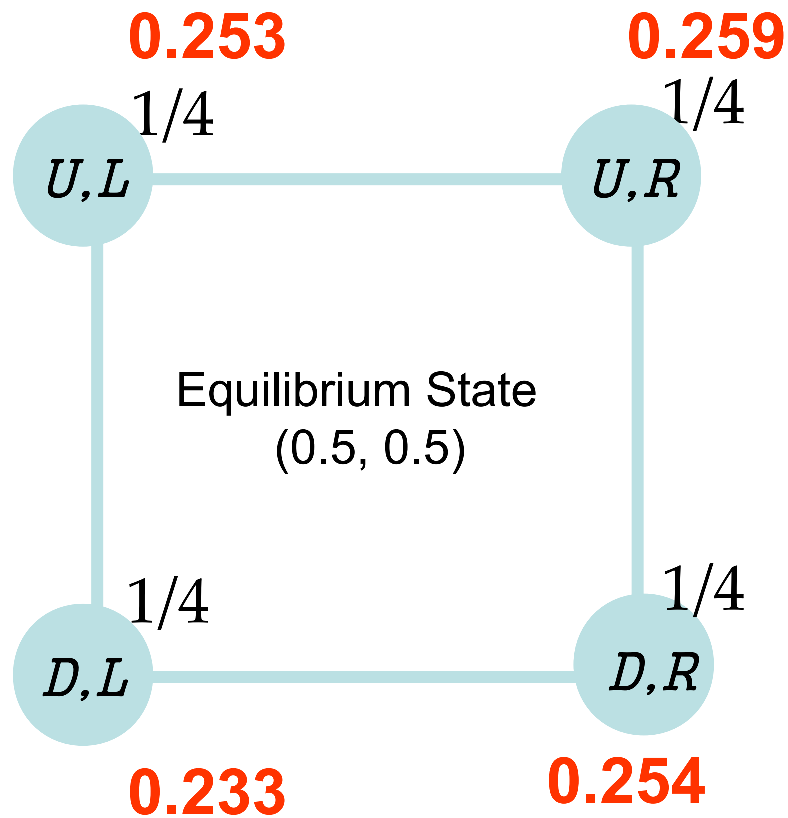
strategy possibility
0.5



What we have known in matching pennies game?

		1/2	1/2
		L	R
1/2	U	5, 0	0, 5
1/2	D	0, 5	5, 0

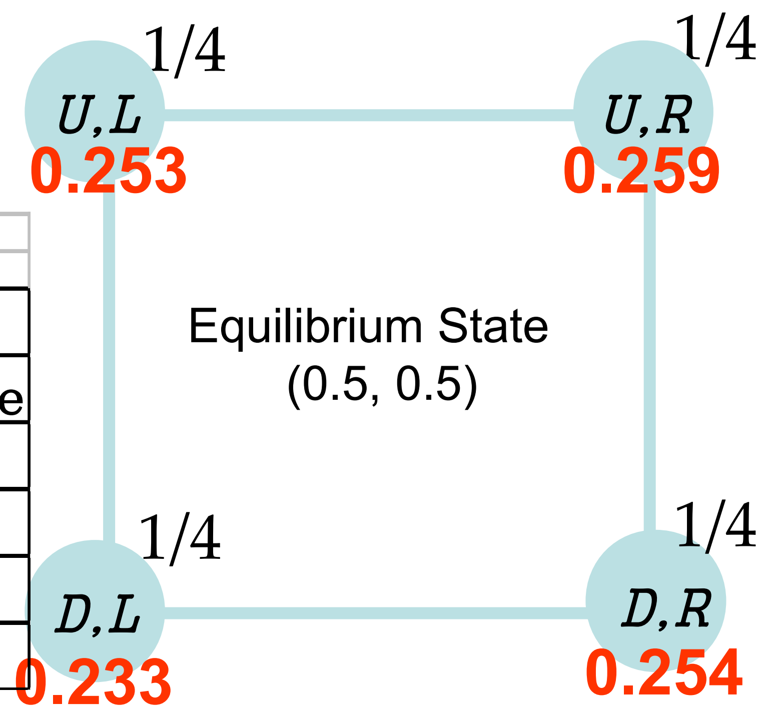
strategy possibility
0.5



What we have known in matching pennies game?

		1/2	1/2
		L	R
1/2	U	5, 0	0, 5
1/2	D	0, 5	5, 0

	Person	Mechine	Mechine	Person
	Person	Mechine	Person	Mechine
D, L	0. 233	0. 249	0. 256	0. 230
D, R	0. 254	0. 253	0. 246	0. 231
U, L	0. 253	0. 257	0. 250	0. 272
U, R	0. 259	0. 241	0. 248	0. 267



Interface

Input

Output

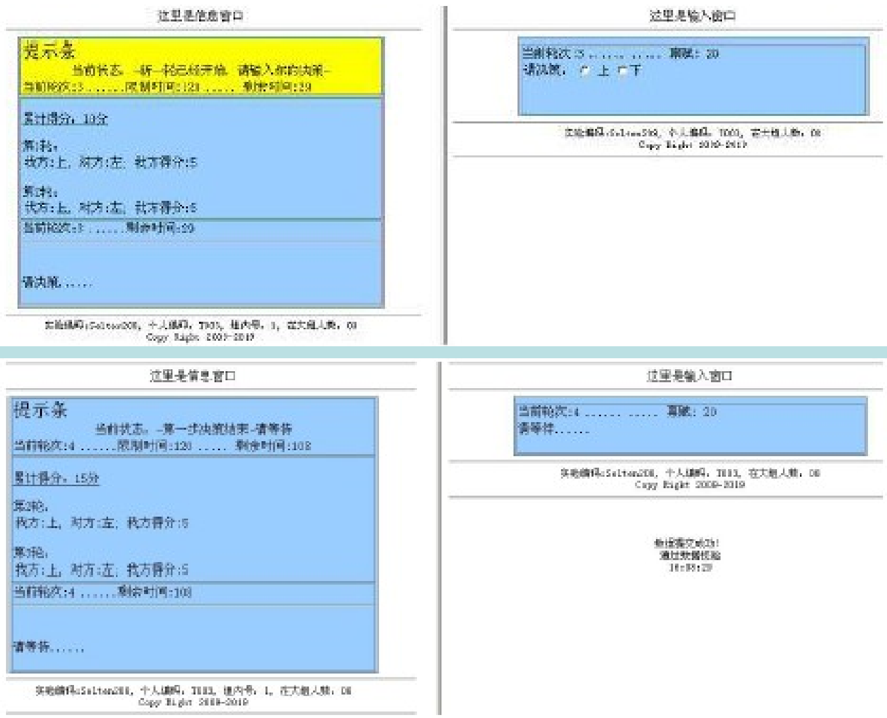


Figure 2: 实验界面图(1)输入界面；(2)响应界面